International Journal of Informatics and Data Science Research
ISSN 2997-3961 (Online)
Vol. 2, No. 8, Aug 2025,
Available online at: https://scientificbulletin.com/index.php/IJIDSR

Dynamics of the Transient Process Caused by a Change in Flow Rate at the End of a Pipe Section for the Case with and Without Taking into Account the Resistance Force

Sh. Sh. Khojikulov
Institute of Mechanics and Seismic Stability of Structures named after M.T. Urazbayev, Uzbekistan
xojiqulovshaxzod@mail.ru

Article information:
Manuscript received: 4 Jun 2025; **Accepted:** 10 Jul 2025; **Published:** 26 Aug 2025

Abstract: Within the framework of the non-conservative form of quasi-one-dimensional linearized equations of N.E. Zhukovsky, the problem of the transient process in an elementary relief section of a pipeline with a temporary change in the boundary velocities of an incompressible fluid is considered. The incomplete telegraph equation, composed for the average flow velocity, is solved by the method of separation of variables. The pressure is determined by integrating the original equations using the expression found for the velocity.

Numerical results are given for the case of constant values of initial and boundary conditions. Modes with constant, increasing and decreasing average pressure values in the considered section are identified. Peculiarities of shock wave behavior are noted both with and without taking into account the friction force.

Keywords: pipeline, low-compressibility fluid, velocity, pressure, resistance force, gravity, Fourier method, computational experiment.

Introduction

A non-stationary problem of the state of a slightly compressible liquid is formulated and solved, when the change in the density of the liquid and the deformability of the thin wall are taken into account when forming the propagation velocity of small pressure disturbances according to N.E. Zhukovsky, and the initial distributions of the velocity and pressure are known. The path change in pressure is due to a non-conservative form of the local component of the inertial force and the resistance force, which is linearized. The temporary change in pressure is due to the velocity gradient over distance.

In engineering practice, hydraulic calculations of pipelines and their networks are usually carried out based on the characteristics of a steady flow, remote from disturbance zones [1]. However, when considering non-stationary processes, so-called "bottlenecks" of the pipeline system are revealed. A typical example of a non-stationary problem is the problem of transient processes. The transition from one steady state to another is usually accompanied by phenomena that require simultaneous consideration of two characteristic speed scales: the hydrodynamic flow velocity and the propagation velocity of small disturbances in the "pipe - transported medium" system. The formation and propagation of disturbance waves in pipelines are studied within the framework of N.E. Zhukovsky's quasi-one-dimensional

equations [2, 3]. He was the first to construct a system of quasi-one-dimensional equations that simultaneously take into account the hydrodynamic velocity of the fluid medium and the propagation velocity of small disturbances in the medium-pipeline system, and conducted theoretical and experimental studies on the propagation of compaction and rarefaction waves in pipelines.

Problems of propagation of compaction and rarefaction waves are of particular interest for science and practice. Multiple repetitions of such waves lead to loss of pipeline integrity. Let us give two examples that demonstrate the destructive consequences of repetitions of alternating compaction and rarefaction waves.

The start-up and shutdown of pumping station units is accompanied by the formation of pressure disturbance waves in large, destructive intervals [4].

In general, the study of the characteristics of the formation and propagation of shock waves is of great interest. The difficulty of the process of studying shock waves is associated with jumps in indicators. This applies to both theoretical studies and bench studies. In the first case, this is due to the use of the Heaviside function [5] or the introduction of traveling waves and the development of adequate numerical methods [6, 7]. In the second case, this is due to the development and use of less inertial measurement systems with the support of modern computer technologies [8, 9].

Based on these features of the shock wave and the needs of engineering practice for research, an elementary relief section of a pipeline is considered below, at both ends of which the laws of change in velocity over time are specified. The initial distributions of velocity and its derivative over time are known. It is necessary to solve the problem by the Fourier method within the framework of the quasi-one-dimensional model of N.E. Zhukovsky.

A general solution to the problem is proposed taking into account the force factors of friction, gravity and the local component of the inertial force at variable input and output velocities. Numerical results are obtained for constant values of the functions appearing in the initial and boundary conditions. Unlike known solutions, the relief of the pipeline route is taken into account here.

Statement and method of solving the problem

The dynamic state of the liquid in an elementary section of a pipeline is $\rho \approx const$ described by the equations [2, 10]:

$$\begin{cases} -\dfrac{\partial P}{\partial x} = \rho\left(\dfrac{\partial w}{\partial t} + 2aw\right), \\ -\dfrac{\partial P}{\partial t} = \rho c^2 \dfrac{\partial w}{\partial x}. \end{cases} \qquad (1)$$

Here $P(x,t) = p(x,t) + \rho g y(x)$ is the total hydrostatic pressure, which represents a measure of the energy loss of the liquid; $p(x,t),\ w(x,t)$ are the average x values of pressure and flow velocity over the cross-section at the moment of time t; $y(x)$ is the leveling height of the pipeline axis; $a = \dfrac{\lambda w_*}{4D};$ w_* – characteristic speed of the process –

averaging parameter; λ – coefficient of friction resistance; $c = \left(\frac{\rho_0}{k} + \frac{D\rho_0}{E\delta} \right)^{-1/2}$ – speed of propagation of small pressure disturbances in the pipe-liquid system; $\rho_0,\ D$ – density of the liquid and diameter of the pipeline in an undisturbed state; $k,\ E$ – elastic moduli of the transported liquid and pipe; δ – pipe thickness [10].

Elimination of full pressure $P(x,t)$ at constant cross-sectional area of the pipe $f = \pi D^2 / 4$ allows us to construct a separate equation for the velocity:

$$\frac{\partial^2 w}{\partial t^2} + 2a\frac{\partial w}{\partial t} = c^2 \frac{\partial^2 w}{\partial x^2}.$$

This is the incomplete telegraph equation [11].

The following variant of initial conditions is considered:

$$w(x,0) = w^{(0)}(x),\ \frac{\partial w(x,0)}{\partial t} = w^{(t)}(x).$$

The boundary conditions are given in the form

$$w(0,t) = \overline{w}(t),\ w(l,t) = \tilde{w}(t).$$

According to these data and the first equation of the original system, the initial condition for the total pressure is

$$P(x,0) = p_{00} - \rho \int_0^x \left[w^{(t)}(\eta) + 2a w^{(0)}(\eta) \right] d\eta,$$

where p_{00} is the initial pressure in the inlet section.

Let us bring the boundary conditions to a homogeneous form. For this purpose, a new desired function is introduced $u(x,t)$ according to the dependence

$$w(x,t) = C_0(t) + xC_1(t) + u(x,t).$$

The values of the functions are determined

$$C_0(t) = \overline{w}(t),\ C_1(t) = \frac{\tilde{w}(t) - \overline{w}(t)}{l},$$

which allow us to represent the boundary conditions in a homogeneous form:

$$u(0,t) = 0,\ u(l,t) = 0.$$

In this case, the equation itself becomes non-homogeneous.

$$\frac{\partial^2 u}{\partial t^2} + 2a\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} - H(t) - xR(t).$$

It is solved under new initial conditions.

$$u(x,0) = w^{(0)}(x) - C_0(0) - C_1(0)x,$$

$$\frac{\partial u(x,0)}{\partial t} = w^{(t)}(x) - C_0'(0) - C_1'(0)x$$

and under already known homogeneous boundary conditions.

From here on we use the notations

$$H(t) = C_0''(t) + 2aC_0'(t), \quad R(t) = C_1''(t) + 2aC_1'(t).$$

Let us proceed to solving the equation with respect to the auxiliary function. The equations and boundary conditions are linear. This facilitates the process of solving the problem, and we seek the solution as the sum of the superposition of two solutions [10, 11].

The first term $U(x,t)$ is the general solution of the homogeneous equation

$$\frac{\partial^2 U}{\partial t^2} + 2a\frac{\partial U}{\partial t} = c^2\frac{\partial^2 U}{\partial x^2}$$

at given boundary conditions $U(0,t) = 0,\ U(l,t) = 0$ and initial conditions

$$U(x,0) = w^{(0)}(x) - C_0(0) - C_1(0)x,$$

$$\frac{\partial U(x,0)}{\partial t} = w^{(t)}(x) - C_0'(0) - C_1'(0)x.$$

The second term $V(x,t)$ represents a particular solution of the inhomogeneous equation in accordance with the right-hand side

$$\frac{\partial^2 V}{\partial t^2} + 2a\frac{\partial V}{\partial t} = c^2\frac{\partial^2 V}{\partial x^2} - H(t) - xR(t),$$

which is solved with zero initial values

$$V(x,0) = 0, \quad \frac{\partial V(x,0)}{\partial t} = 0.$$

and homogeneous boundary conditions

$$V(0,t) = 0,\ V(l,t) = 0.$$

First, the homogeneous equation is solved.

Let's assume that

$$U(x,t) = X(x)Y(t),$$

and we arrive at the equations:

$$\frac{Y''(t) + 2aY'(t)}{c^2 Y(t)} = \frac{X''(x)}{X(x)} = -\lambda^2.$$

The Sturm-Liouville problem for the coordinate x gives [10 ,1 1 ,1 2]

$$X_n(x) = \sin \lambda_n x,\ \lambda_n = \frac{n\pi}{l},\ \|X_n(x)\|^2 = \frac{l}{2}.$$

In this case, to determine the multiplier in the form of an eigen function of time, we have the equation

$$Y_n''(t) + 2aY_n'(t) + c^2\lambda_n^2 Y_n(t) = 0. \qquad (2)$$

Substituting the expression $Y_n(t) = e^{s_n t}$ into this equation will yield the characteristic equation

$$s_n^2 + 2as_n + c^2\lambda_n^2 = 0.$$

The discriminant of this equation is

$$D_n = a^2 - c^2\lambda_n^2,$$

and the solutions of the quadratic equation will be:

$$(s_n)_{1,2} = -a \pm \sqrt{D_n}.$$

Taking these possible options into account $D_n > 0,\ D_n = 0$ and $D_n < 0$ we $Y_n(t)$ get the solution:

$$Y_n(t) = \begin{cases} e^{-at}\left(A_n ch\sqrt{D_n}t + B_n sh\sqrt{D_n}t\right) \text{ при } D_n > 0, \\ e^{-at}\left(A_n + B_n t\right) \text{ при } D_n = 0, \\ e^{-at}\left(A_n \cos\sqrt{|D_n|}t + B_n \sin\sqrt{|D_n|}t\right) \text{ при } D_n < 0. \end{cases}$$

Accordingly, the general solution of the homogeneous equation is

$$U(x,t) = \sum_{n=1}^{\infty} Y_n(t)\sin\lambda_n x.$$

We proceed to the implementation of the boundary conditions of the general solution of the homogeneous equation.

From the first initial condition (at $t = 0$) it follows:

$$\sum_{n=1}^{\infty} A_n \sin\lambda_n x = w^{(0)}(x) - C_0(0) - xC_1(0).$$

We multiply the sides of this equation by $\sin\lambda_m x dx$ and integrate from zero to l. As a result, we get

$$A_n\left\|X_n(x)\right\|^2 = W_n - C_0(0)S_n - C_1(0)K_n,$$

Where

$$W_n = \int_0^l w^{(0)}(x)\sin\lambda_n x dx,$$

$$S_n = \int_0^l \sin \lambda_n x dx = \frac{l}{n\pi}\left[1-(-1)^n\right],$$

$$K_n = \int_0^l x \sin \lambda_n x dx = \frac{l^2}{n\pi}(-1)^{n+1}.$$

From here we determine

$$A_n = \frac{2}{l}\left[W_n - C_0(0)S_n - C_1(0)K_n\right].$$

The second initial condition is related to the $U(x,t)$ time derivative of the function at $t=0$.

We find

$$Y_n'(t) = \begin{cases} e^{-at}\left[\left(-aB_n + \sqrt{D_n}A_n\right)sh\sqrt{D_n}t + \left(-aA_n + \sqrt{D_n}B_n\right)ch\sqrt{D_n}t\right] \text{ при } D_n > 0, \\ e^{-at}\left[-a\left(A_n + B_n t\right) + B_n\right] \text{ при } D_n = 0, \\ e^{-at}\left[\left(-aB_n - \sqrt{|D_n|}A_n\right)\sin\sqrt{D_n}t + \left(-aA_n + \sqrt{|D_n|}B_n\right)\cos\sqrt{D_n}t\right] \text{ при } D_n < 0. \end{cases}$$

Let's calculate

$$Y_n'(0) = \begin{cases} -aA_n + \sqrt{D_n}B_n \ \ при \ \ D_n > 0, \\ -aA_n + B_n \ \ при \ \ D_n = 0, \\ -aA_n + \sqrt{|D_n|}B_n \ \ при \ \ D_n < 0 \end{cases} = -aA_n + \gamma_n B_n,$$

Where $$\gamma_n = \begin{cases} \sqrt{D_n} & \text{при } D_n > 0, \\ 1 & \text{при } D_n = 0, \\ \sqrt{|D_n|} & \text{при } D_n < 0. \end{cases}$$

Then the second initial condition takes the form:

$$\sum_{n=1}^{\infty}(-aA_n + \gamma_n B_n)\sin\lambda_n x = w^{(t)}(x) - C_0'(0) - xC_1'(0).$$

The application of orthonormal eigenfunctions with respect to x, for $W_n^{(t)} = \int_0^t w^{(t)}(\eta)\sin\lambda_n \eta d\eta$, leads to the equality

$$\left(-aA_n + \gamma_n B_n\right)\frac{l}{2} = W_n^{(t)} - C_0'(0)S_n - C_1'(0)K_n.$$

From here we find the values of the coefficients

$$B_n = \frac{a}{\gamma_n} A_n + \frac{2}{\gamma_n l}\left[W_n^{(t)} - C_0'(0) S_n - C_1'(0) K_n \right].$$

The values of the coefficients of the general solution of the homogeneous equation have been determined. Now we move on to solving the non-homogeneous equation.

We believe that

$$V(x,t) = \sum_{n=1}^{\infty} L_n(t) \sin \lambda_n x ,$$

thus we fulfill the boundary conditions $V(0,t) = 0,\ V(l,t) = 0.$

Substituting this solution into the inhomogeneous equation leads to the equation

$$\sum_{n=1}^{\infty} \left[L_n''(t) + 2aL_n'(t) + c^2 \lambda_n^2 L_n(t) \right] \sin \lambda_n x = -H(t) - xR(t) .$$

We use the conditions of orthonormal eigen functions $X_n(x) = \sin \lambda_n x$:

$$L_n''(t) + 2aL_n'(t) + c^2 \lambda_n^2 L_n(t) = \frac{2}{l}\left[-H(t) S_n - R(t) K_n \right]. \quad (3)$$

When $t = 0$ we have the condition $L_n(0) = 0.$ From the second initial condition $\frac{\partial V(x,0)}{\partial t} = 0$ it follows that $L_n'(0) = 0$.

In general, cases are considered where functions $L_n(t)$ are defined analytically. This class includes cases of linear, exponential, trigonometric and some other forms of specifying boundary conditions for velocity. In this case, the final solution for the auxiliary function $u(x,t)$ is defined as

$$u(x,t) = \sum_{n=1}^{\infty} \left[Y_n(t) + L_n(t) \right] \sin \lambda_n x .$$

Going back, we get a solution to the problem in terms of speed

$$w(x,t) = C_0(t) + xC_1(t) + \sum_{n=1}^{\infty} \left[Y_n(t) + L_n(t) \right] \sin \lambda_n x .$$

We find the solution to the problem regarding pressure by integrating the initial system of quasi-one-dimensional equations.

In the first step of the solution, $P(x,t)$ we turn to the second equation $\frac{\partial P}{\partial t} = -\rho c^2 \frac{\partial w}{\partial x}$ of the original system.

We find

$$\frac{\partial w(x,t)}{\partial x} = C_1(t) + \sum_{n=1}^{\infty} \lambda_n \left[Y_n(t) + L_n(t) \right] \cos \lambda_n x .$$

As a result of integrating the second equation of the original system, we obtain the formula for pressure:

$$P(x,t) = -\rho c^2 \left[C_1^{(0)}(t) + \sum_{n=1}^{\infty} \lambda_n \left[Y_n^{(0)}(t) + L_n^{(0)}(t) \right] \cos \lambda_n x \right] + p_t(x).$$

Here

$$\int_0^t C_1(\theta) dt = C_1^{(0)}(t) \quad \text{And} \quad \int_0^t L_n(\theta) d\theta = L_n^{(0)}(t),$$

$$Y_n^{(0)}(t) = \int_0^t Y_n(\theta) d\theta = \begin{cases} \frac{1}{c^2 \lambda_n^2} \left[\left(-A_n a - B_n \sqrt{D_n} \right) \left(e^{-at} ch \sqrt{D_n} t - 1 \right) + \right. \\ \left. + \left(-A_n \sqrt{D_n} - B_n a \right) e^{-at} sh \sqrt{D_n} t \right] \quad \text{при } D_n > 0, \\ A_n \frac{1 - e^{at}}{a} + B_n \left(-\frac{te^{-at}}{a} + \frac{1 - e^{-at}}{a^2} \right) \quad \text{при } D_n = 0, \\ \frac{1}{c^2 \lambda_n^2} \left[\left(-A_n a - B_n \sqrt{|D_n|} \right) \left(e^{-at} \cos \sqrt{|D_n|} t - 1 \right) + \right. \\ \left. + \left(A_n \sqrt{|D_n|} - B_n a \right) e^{-at} \sin \sqrt{|D_n|} t \right] \quad \text{при } D_n < 0. \end{cases}$$

It is necessary to determine the value $p_t(x)$ of the constant of integration over time. To determine it, we turn to the first equation $-\frac{\partial P}{\partial x} = \rho \left(\frac{\partial w}{\partial t} + 2aw \right)$ of the original system.

Let's calculate

$$\frac{\partial P(x,t)}{\partial x} = \rho c^2 \sum_{n=1}^{\infty} \lambda_n^2 \left[Y_n^{(0)}(t) + L_n^{(0)}(t) \right] \sin \lambda_n x + p_t'(x)$$

And

$$\frac{\partial w(x,t)}{\partial t} = C_0'(t) + xC_1'(t) + \sum_{n=1}^{\infty} \left[Y_n'(t) + L_n'(t) \right] \sin \lambda_n x .$$

When $t = 0$ the terms highlighted by a superscript (0) and capital letters are converted to zero. In this connection, to determine the value of the integral, we obtain the equality:

$$p_{00} - \rho \int_0^x \left[w^{(t)}(\eta) + 2aw^{(0)}(\eta) \right] d\eta =$$

$$= -\rho \left[C_0'(t) + 2aC_0(t) \right] x - \rho \left[C_1'(t) + 2aC_1(t) \right] \frac{x^2}{2} + \rho \int_0^x E(\eta) d\eta.$$

From here we find the value of the integral and insert it into the formula for pressure:

$$P(x,t) = p_{00} - \rho\int_0^x \left[w^{(t)}(\eta) + 2aw^{(0)}(\eta) \right] d\eta -$$

$$-\rho c^2 \left\{ C_1^{(0)}(t) + \sum_{n=1}^{\infty} \lambda_n \left[Y_n^{(0)}(t) + L_n^{(0)}(t) \right] \cos \lambda_n x \right\}.$$

$$C_1^{(0)}(t) = \frac{1}{l}\int_0^t \left[\bar{w}(\theta) - \tilde{w}(\theta) \right] d\theta.$$

Here

Thus, the functions for the boundary velocities $\bar{w}(t),\ \tilde{w}(t)$ are twice differentiable and integral functions, and the functions for $w^{(0)}(x)$ and $w^{(t)}(x)$ are integral.

We demonstrated the formal course of solving the problem. In the process of solving the problem with respect to $w(x,t)$ and $P(x,t)$ we considered individual derivatives and integrals to be already known. But in the calculations we limit ourselves to considering only problems about transient processes. For each of the cases we will provide specific values of derivatives and integrals.

Discussion of results

In option I, the calculation was carried out using $w_{00} = 0\ м/с,\quad w^{(0)} = w^{(1)} = 5\ м/с$. That is, in a stationary section under high pressure, $7.0\,МПа$ liquid began to flow at the inlet with a speed $5\ м/с$, and liquid began to be taken from the outlet with a speed $5\ м/с$. The mass of liquid in the section is constant.

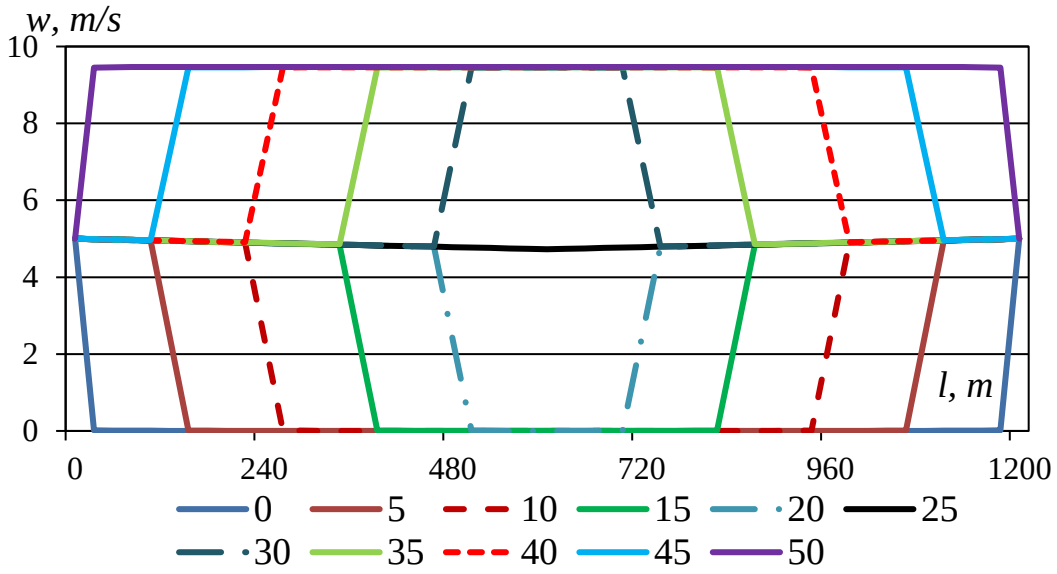

Fig. 1. Graphs of speed along the length of the section, obtained in the first ten time steps with an interval of $l/(10c)$. $l = 1200\ м,\ D = 0.2\ м,\ \lambda = 0.018,\ w_* = 5\ м/с,$ $c = 1200\ м/с,\ w_{00} = 0\ м/с,\ w^{(0)} = w^{(1)} = 5\ м/с,\ p_{00} = 7.00\ МПа$

Fig. 1 shows the velocity curves at different time steps of the first conditional period.

In this variant, the conditional period of the process relative to the indicators is equal to l / c. The lower curves (at $tc / (10l) = 0,\ 5,\ 10,\ 15,\ 20$) refer to the first half of the conditional period. The process of increasing the speed begins simultaneously from both ends of the section: from zero to $5\ м / с$. The waves gradually move towards each other. At $tc / (10l) = 25$ they mutually meet, forming an almost straight line with a constant speed $5\ м / с$. In this case, approaching the middle of the section occurs with the loss of a certain speed. Adding up these pulses, they bring the speed to a new value, which is already less than $2 \cdot 5 м / с = 10 м / с$. After this, the waves of compaction and rarefaction continue their movements until they reach the ends of the section. But again, the values of the speed jump decrease by a certain value.

Having reached the ends of the section, the waves turn over and strive back to a mutual meeting. Unlike the first four lower graphs, here in the lower links the speed is no longer zero, but approximately unity. When the waves return, the process is repeated, but with a smaller maximum value. And so on. In odd conditional periods, the upper envelope line of the pressure curves is transformed, and in even ones, the lower envelope. This judgment is also confirmed by the graphs in Fig. 2, which present the results of the change in speed over time for sections $x = 24,\ 600, 912$ и $1176\ м$. Here one can observe the nature of the gradual transition to a regime with a constant speed $5\ м / с$.

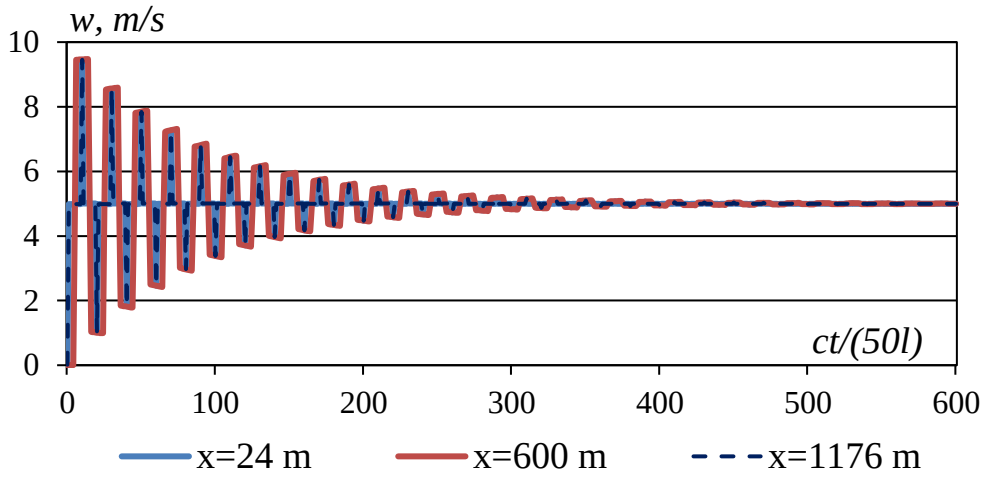

Fig. 2. Flow velocity graphs in different sections of the site.

For data see Fig. 1

When using a larger time interval (Fig. 3), an exponential decrease in the pressure amplitude is observed. In this case, the limiting pressure values at the ends and in the middle of the section are consistent with the new flow velocity values. In the middle of the section, the value that was adopted at the beginning of the process is established.

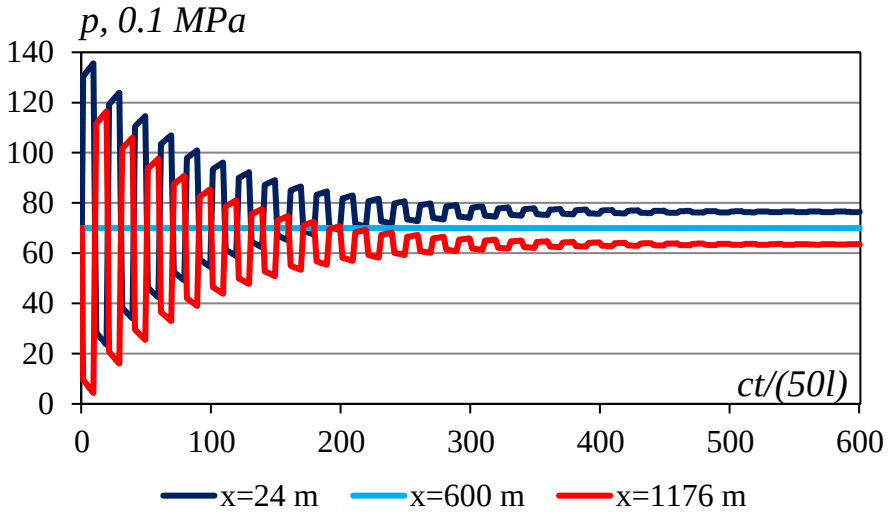

Fig. 3. Change in pressure over time in different sections of the pipeline. Data see Fig. 1

In all variants, periodic solutions were obtained with respect to the flow velocity. For , $w^{(0)} \neq w^{(1)}$ solutions with a period were obtained $2c/l$, and for $w^{(0)} = w^{(1)}$ – with a period c/l. In the case , $a \neq 0$ these values constituted a conditional period expressing the nature of the change in the indicators in the section. In this case, the steps in the velocity graphs, separated by velocity jumps and boundary conditions, turned out to be strictly horizontal. That is, the interval of velocity change during jumps remains the same during the propagation of one-sided disturbances. When compaction and rarefaction waves formed by identical boundary velocities meet, instantaneous mutual compensation and overturning of the waves are observed, which continue to propagate further without losses (Fig. 4).

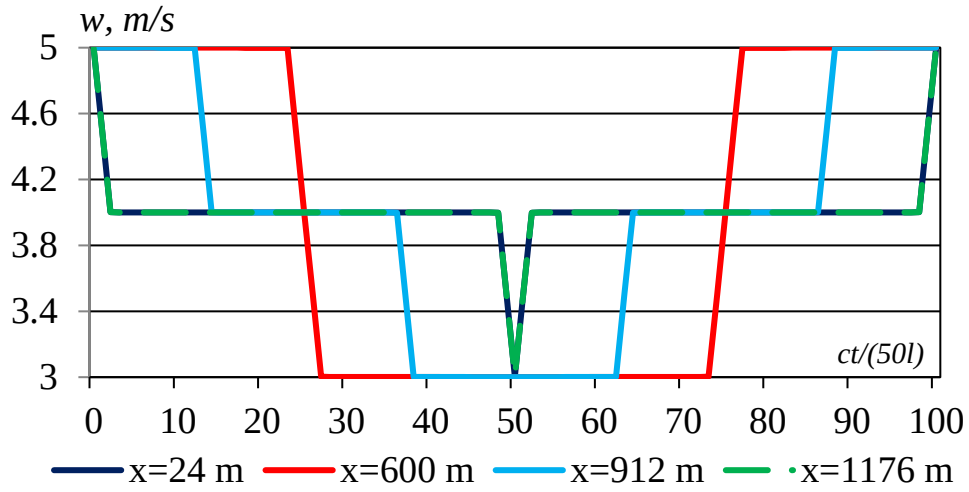

Fig. 4. Graphs of speed over time in different sections of the site at $a = 0.$

$$w_{00} = 5\ м/с,\ w^{(0)} = w^{(1)} = 4\ м/с$$

As can be seen from the figure, when moving to a new speed value on both sides of the section, the speed change interval is strictly . The amplitude of speed changes for one-sided disturbances is in the interval with length. Typical graphs for such a case are shown in Fig. 5.

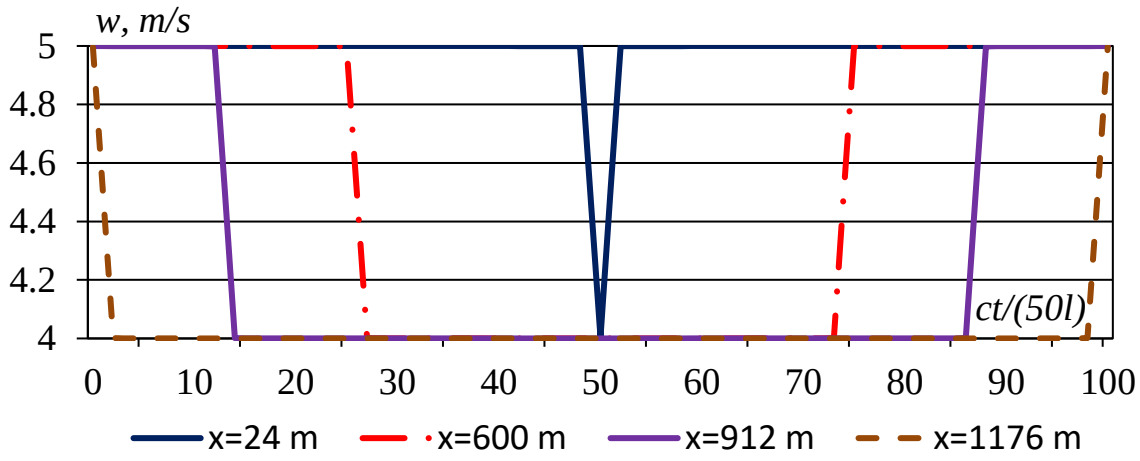

Fig. 5. Changes in flow velocity over time in one period in different sections. $a = 0$, $w_{00} = w^{(0)} = 5\ м/с$, $w^{(1)} = 4\ м/с$

The constancy of the values in the graph steps remains in force for the results of the pressure calculation. The periodicity of the pressure solution is observed when there is a general increase in pressure at the end of each period (Fig. 6).

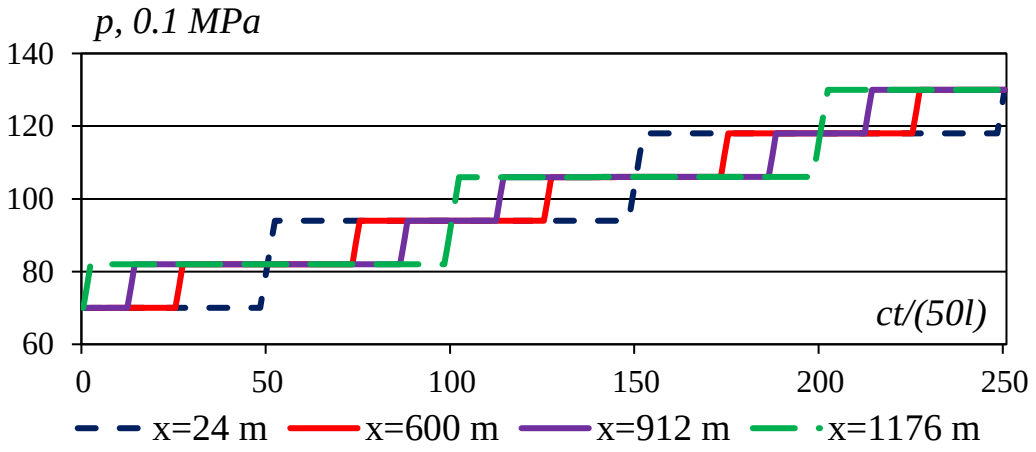

Fig. 6. General increase in pressure at $w_{00} = w^{(0)} = 5\ м/с$, $w^{(1)} = 4\ м/с$

When the opposite effect is observed. But in both cases, when the term with the multiplier is excluded, the same solution is obtained for individual periods.

Conclusion

The solution to the problem for flow velocity was obtained by the Fourier method in the form of a functional series, and the solution for pressure was obtained by integrating the initial equations with a known solution for velocity.

For cases where the functions in the boundary conditions have constant values, a calculation program was compiled and a computational experiment was carried out with and without taking into account the frictional resistance force.

Variants of transient processes under conditions of a linearized quadratic law of resistance are studied, when the mass of liquid in the section is constant, linearly increasing and linearly decreasing depending on time. Features of the propagation and reflection of one-

and two-sided velocity disturbances in an elementary section of a pipeline are revealed, tending to reconstruct the velocity and pressure fields according to new boundary conditions with the formation of unique envelope curves.

Using the found velocity value and the initial equations, a solution to the problem for the total pressure is obtained, which in the case of a horizontal pipeline is converted to normal pressure. In the case of a positive difference between the input and output velocities, a monotonic increase in pressure in the section is expected, and with a negative difference - a monotonic decrease. The rate of change of the average pressure in an elementary section is proportional to the difference between the input velocity. In this regard, when solving the problem makes sense only for small time intervals.

The results of the work were obtained for a developed turbulent flow regime and a circular cross-section of the pipeline. To apply the results when calculating a pipeline with a different cross -section shape, instead of the diameter, one can use the double equivalent radius of the pipeline.

Literatures

1. Lewandowski A. New Numerical Methods For Transient Modeling of Gas Pipeline Networks. – New Mexico: Pipeline Simulation Interest Group, 1995.
2. Charny I.A. Unsteady motion of real fluid in pipes. 2nd ed. – M.: Nedra, 1975. – 296 p.,
3. Seleznev V.E., Aleshin V.V., Pryalov S.N. Modern computer simulators in pipeline transport. Mathematical methods of modeling and practical application / Ed. V.E.Seleznev. – M.: MAKS Press, 2007. – 200 p.
4. Rajabov U.M., Zhonkobilov U.U. Calculation of hydraulic shock absorbers in long pressure pipelines of pumping stations. - Tashkent: Intellect nashriyot. 2022. - 130 p.
5. Bobrovsky S.A., Shcherbakov S.G., Huseynzade M.A. Gas movement in gas pipelines with path selection. – M.: Nauka, 1972. – 192 p.
6. Bozorov O.Sh., Mamatkulov M.M. Analytical studies of nonlinear hydrodynamic phenomena in media with slowly changing parameters. – Tashkent, TITLP, 2015. – 96 p.
7. Ermolaeva N.N. Mathematical modeling of non-stationary non-isothermal processes in moving multiphase media: Diss.... Doctor of Physical and Mathematical Sciences. – St. Petersburg, 2017. – 323 p.
8. Aline Barbosa Figueiredo, Renan Martins Baptista, Felipe Bastos de Freitas Rachid, Gustavo Cesar Rachid Bodstein. Numerical simulation of stratified-pattern two-phase flow in gas pipelines using a two-fluid model // International Journal of Multiphase Flow 2017. Vol. 88. –pp. 30-49. DOI 10.1016/j.ijmultiphaseflow.2016.09.016
9. Zemenkov Yu.D., Shalay V.V., Zemenkova M.Yu. Immediate analyzes and calculation of saturated steam pressure of gas condensates for transportation conditions // Procedia Engineering, 2015. Vol. 113. – pp. 254-258.
10. Budak B.M., Samarskii A.A., Tikhonov A.N. Collection of problems in mathematical physics. - M.: Nauka, 1972. - 678 p., Charny
11. Tikhonov A.N., Samarsky A.A. Equations of mathematical physics. – M.: Nauka, 1977. – 736 p., Budak
12. Khozhaev I.K., Khozhukov Sh.Sh., Shamsutdinova N.Sh. The Sturm-Liouville problem for an arbitrary combination of inhomogeneous boundary conditions of the first, second

and third kinds of boundary conditions // Problems of Computational and Applied Mathematics, 2023, No. 2(47). – P. 125-128.

13. Kamke E. Handbook of ordinary differential equations. – Moscow: Nauka, 1976. – 576 p.
14. Bronstein I.N., Semendyaev K.A. Handbook of Mathematics. – M.: Nauka, 1964. – 608 p.
15. Shterenlicht D.V. Hydrasulics. Textbook for Universities. – M.: Energoatomizdat, 1991. – 351.