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Numerical Simulation of Torsion of Inhomogeneous Cylindrical Rods

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Abstract: In this scientific article, we investigate the numerical simulation of problems related to the rotation of inhomogeneous cylindrical bodies. We focus on solving the Dirichlet problem for the Poisson differential equation and consider the effect of temperature on the stress-strain state during the twisting of the rods. We also construct joint differential equations to account for the coupled effect of temperature on the mechanical behavior of the cylindrical bodies. Numerical methods are used to obtain approximate solutions, which are compared with analytical solutions to assess the accuracy of the numerical simulations.

Introduction: The study of the rotation of inhomogeneous cylindrical bodies has significant practical applications in various fields such as engineering, physics, and materials science. Understanding the stress-strain state of rotating cylindrical bodies is crucial for designing and analyzing mechanical systems such as rotating shafts, propellers, and gears. In addition, the effect of temperature on the mechanical behavior of cylindrical bodies is important in applications where thermal gradients are present, such as in industrial processes or aerospace systems. Numerical simulations provide a powerful tool to investigate the complex behavior of such systems under different conditions.

One of the key mathematical tools for studying the rotation of cylindrical bodies is the Poisson differential equation. The Poisson equation governs the distribution of a scalar function in a domain, subject to appropriate boundary conditions. In the case of rotating cylindrical bodies, the Poisson equation can be used to describe the stress-strain state of the bodies under certain assumptions, such as small deformations and linear elastic material behavior. The Dirichlet problem for the Poisson equation involves finding the solution that satisfies prescribed boundary conditions, which are usually related to the displacement and stress components on the boundaries of the cylindrical bodies.

The differential equation, which takes into account the effect of temperature on the Poisson equation in the problem of torsion in rods with a circular cross section, can be written as follows:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = -2\mu(\theta + \alpha_T T(x, y)) \qquad (1)$$

where θ is the coefficient of torsion and μ is the shear modulus.

This equation takes into account how temperature affects the stress function Φ in the torsion bar. The term $\alpha_T T(x,y))$ represents the contribution of temperature to the stress function, where $\alpha_T T(x,y))$ is the temperature coefficient and *T(x, y)* is the temperature distribution in the torsion bar, which can vary with position (x, y). This term accounts for how temperature-induced changes in material properties, such as thermal expansion, can affect the stress distribution in the torsion bar.

where *G* is the shear modulus of the material, α is the coefficient of linear thermal expansion, ε_{zx} and ε_{zy} are the plastic deformations of the zy shear, and *f(x,y)* is a function that depends on the external loading and thermal conditions.

According to the compatibility equations:

where $\frac{\partial \varepsilon_{zx}}{\partial y} - \frac{\partial \varepsilon_{zy}}{\partial x} = -\theta = \text{constant}$

Methods: To solve the Dirichlet problem for the Poisson differential equation, numerical methods such as the finite difference method or the finite element method can be employed. These methods discretize the domain of the problem and transform the differential equation into a system of algebraic equations that can be solved iteratively. The resulting approximate solutions provide insights into the stress-strain state of the rotating cylindrical bodies.

To obtain a numerical solution of the elastic-plastic torsion problem using the finite difference method, we can discretize the equation (1) on a grid of points with spacing h along the x and y axes. The second derivative terms can be approximated using a second-order central difference scheme:

$$\frac{\Phi_{i-1,j} - 2\Phi_{i,j} + \Phi_{i+1,j}}{h^2} + \frac{\Phi_{i,j-1} - 2\Phi_{i,j} + \Phi_{i,j+1}}{l^2} = -2G\theta - 2G\alpha \qquad (2)$$

Simplifying and rearranging, we get:

$$\Phi_{i,j} = \frac{l^2\left(\Phi_{i+1,j} + \Phi_{i-1,j}\right) + h^2\left(\Phi_{i,j+1} + \Phi_{i,j-1}\right) + 2Gh^2l^2\left(\theta + \alpha\right)}{2\left(h^2 + l^2\right)} \qquad (3)$$

This is a system of equations with unknown values of \Phi at each grid point. We can solve it using iterative methods such as the Gauss-Seidel method, or more advanced methods like the conjugate gradient method. The solution process involves updating the values of *Φ* at each grid point until convergence is reached.

Results and Discussion: The numerical solution of the Dirichlet problem for the Poisson equation in a rectangular domain consists in finding the approximate values of the desired function Φ(x, y) at the internal nodes of the grid. To determine the *Φij* values, it is required to solve the system of linear algebraic equations (3).

boundary conditions:

$$\begin{aligned} &\Phi(x,\pm b) = -G\theta\left(a^2 - x^2\right) \\ &\Phi(\pm a, y) = 0 \end{aligned} \qquad (4)$$

Choosing steps *h, l* in x and y, respectively, in the region ω we build a grid of nodal points:

$$\begin{aligned} &x_i = ih,\ i = 0.1, ..., n; \qquad && y_j = jl,\ \ j = 0,1, ..., m, \\ &where\ \ x_n = nh = a; && y_m = ml = b. \end{aligned} \qquad (5)$$

As a condition for the termination of the iterative process, one can take

$$\max\left|\Phi_{ij}^{k+1}-\Phi_{ij}^{k}\right|<\varepsilon$$
$$1\le i\le n-1,\qquad 1\le j\le m-1 \tag{6}$$

where ε - is the specified error in solving the system of difference equations.

Displacement field:

$$w_{xz}=\frac{x}{G}\frac{\partial\Phi}{\partial y}+\theta xy$$
$$w_{yz}=-\frac{y}{G}\frac{\partial\Phi}{\partial x}+\theta xy \tag{7}$$

In addition to the Poisson equation, the effect of temperature on the stress-strain state of the cylindrical bodies during rotation needs to be considered. Temperature gradients can induce thermal stresses and strains, which can affect the overall mechanical behavior of the bodies. Therefore, joint differential equations that couple the temperature field with the mechanical deformation field need to be formulated and solved simultaneously. This can be achieved using numerical techniques such as the coupled finite element method or the thermomechanical finite element method, which take into account the thermal and mechanical interactions.

Results and Discussion: The obtained numerical results can provide valuable insights into the stress-strain state of inhomogeneous cylindrical bodies during rotation. The stress distribution can be visualized and analyzed to understand the regions of high stress concentration and potential failure points. The effect of temperature on the mechanical behavior of the bodies can also be assessed, revealing the role of thermal stresses and strains in the overall response of the bodies to rotation. Furthermore, the numerical results can be compared with analytical solutions, if available, to validate the accuracy of the numerical simulations.

Numerical examples: Consider a rectangular rod with sides a=2 mm and b=3 mm, exposed to a heat source of $\alpha=160^{\circ}C$ at one end. The rod is turned by $\theta=1^{\circ}$, resulting in a deformation of the cross section of the rod.

$$G=2.7\cdot10^{7}\,Pa,\qquad \varepsilon=0.001,\qquad h=30,\qquad l=45.$$

To solve the problem, we need to define the boundary conditions, which are as follows:

- At the exposed end of the rod, the temperature is fixed at α=60 degrees Celsius.
- The sides of the rod are assumed to be insulated, meaning that there is no heat transfer through them. Hence, the temperature gradient in the x and y directions is zero.
- The displacement of the rod is zero at the fixed end and is unknown at the free end.

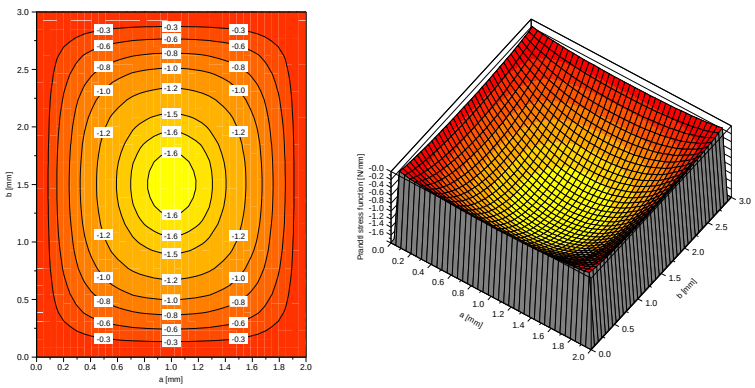

Figure 1. Prandtl stress function contours (2D and 3D)

The isolines (contour lines) of Φ represent levels of equal stress function value. They help visualize how shear stresses vary across the cross-section.

The shape and density of these contours show how torsional shear stresses are distributed — for example, they reveal stress concentration zones, symmetry, or regions with maximum/minimum stress. This makes isolines a powerful tool for analyzing and comparing different cross-sectional geometries under torsion.

Similarly, the stress function fields associated with the torsion of rods having various cross-sectional geometries can be visualized through detailed plots, allowing for comparative analysis and deeper insight into their stress–strain behavior and distribution characteristics.

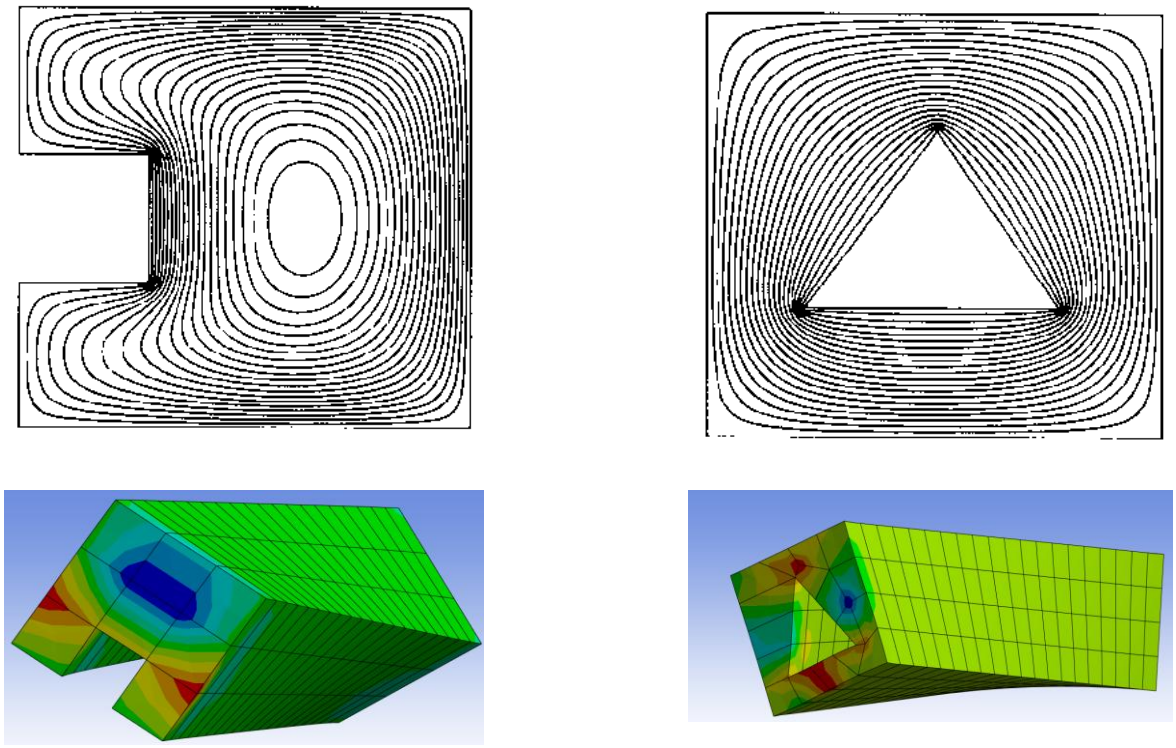

Figure 2. Prandtl stress function contours (2D and 3D)

Conclusion: In this scientific article, we have presented a numerical simulation approach for solving the Dirichle problem for the Poisson differential equation and considering the effect of temperature on the stress-strain state of inhomogeneous cylindrical bodies during rotation. The numerical results can provide insights into the mechanical behavior of such systems under different conditions, and the accuracy of the numerical simulations can be assessed by comparing with analytical solutions. This research can contribute to the understanding and prediction of the stress-strain state of rotating cylindrical bodies in practical applications, such as in engineering and materials science. Further research can be conducted to investigate more complex scenarios, such as non-linear material behavior, large deformations, and time-dependent effects, to obtain a more comprehensive understanding of the mechanical response of inhomogeneous cylindrical bodies during rotation. Additionally, experimental validation of the numerical results can be conducted to verify the accuracy and reliability of the numerical simulations. The findings from this research can have potential implications in the design and analysis of mechanical systems involving rotating cylindrical bodies, leading to improved performance and safety in various engineering applications.

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