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# Mathematical Model of Dust Particle Dispersion Based on Air Temperature And Humidity

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**Annotation**
This article addresses the dispersion of dust particles influenced by temperature and humidity using an extended advection-diffusion model. Dust particles suspended in the air pose significant risks to public health and the environment, especially in industrial or densely populated urban areas. The proposed model incorporates temperature- and humidity-dependent velocity and diffusion coefficients, along with a source term that captures emission or absorption processes. By applying standard partial differential equation theory and well-known numerical methods, the model is shown to be well-posed under reasonable assumptions. Simulations demonstrate that higher temperatures tend to accelerate dust transport, while increased humidity can slow the diffusive spreading or promote particle aggregation. Although real-world scenarios often involve more complex, nonlinear interactions, these findings provide a foundational understanding that can be refined through experimental validation. Future research aims to incorporate gravitational settling, turbulence, and more detailed particle characteristics, ultimately supporting the design of air pollution mitigation strategies and ventilation systems.

**Keywords:** Dust dispersion, advection-diffusion model, temperature and humidity effects, environmental modeling, MATLAB simulation.

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## 1. Introduction

Nowadays, the impact of dust particles on the environment, especially air pollution around industrial areas, climate change, and human health issues, has become extremely relevant. Dust particles (aerosols), suspended in the air, can significantly affect human health by entering the respiratory system. The dispersal of dust particles in the atmosphere depends on multiple factors, among which ambient temperature and humidity play crucial roles. This article aims to analyze the dispersion of dust particles under varying temperature and humidity conditions using a mathematical model.

The proposed mathematical model takes into account volumetric diffusion, advection (transport), and thermodynamic properties of dust particles, as well as changes caused by humidity. This model is particularly beneficial in practical domains such as environmental monitoring in large cities or industrial zones, protecting public health, and designing ventilation systems.

## 2. Materials and Methods

The fundamental equations describing the dispersion of dust particles in the air are the advection-diffusion equations. Let C(r,t) represent the concentration (or mass density) of dust particles at location r in space at time t.

If we examine the time evolution of this concentration, the general advection-diffusion equation is:

$$\frac{\partial C}{dt} + \nabla(uC) = \nabla \cdot (D\nabla C) + S(r,t)$$

where:

- ✓ $u$ is the velocity vector of the air,
- ✓ $D$ is the diffusion coefficient,
- ✓ $S(r,t)$ represents source (emission) or sink (absorption) terms.

Additionally, dust particles' dispersion can depend on temperature and humidity. It is well known that higher humidity can lead to particle agglomeration, thus altering their dispersion properties, while higher temperature can increase the kinetic energy of particles. Therefore, for a more comprehensive model, temperature T and humidity H need to be incorporated.

Temperature T(r,t) and humidity H(r,t) are time-dependent variables that also vary spatially. To reflect their effect on the concentration of dust particles, we consider the following extended advection-diffusion equation:

$$\frac{\partial C}{dt} + \nabla(u(T,H)C) = \nabla \cdot (D(T,H)\nabla C) + S(r,t,T,H)$$

where:

- ✓ $u(T,H)$ is the air velocity, which depends on temperature and humidity,
- ✓ $D(T,H)$ is the diffusion coefficient, also dependent on temperature and humidity,
- ✓ $S(r,t,T,H)$ is the source or sink term that may change according to temperature and humidity.

Temperature and humidity directly affect air density and the overall flow characteristics, influencing the movement of particles. As a simple linear approximation:

$$u(T,H) = u_0(1 + \alpha_T(T - T_0) + \alpha_H(H - H_0))$$

where $u_0$, $T_0$, and $H_0$ are reference values for velocity, temperature, and humidity, respectively, while $\alpha_T$ and $\alpha_H$ are linear coefficients describing how velocity changes with temperature and humidity.

The diffusion coefficient may increase with higher temperature and change with humidity. A linear approximation can be:

$$D(T,H) = D_0(1 + \beta_T(T - T_0) + \beta_H(H - H_0))$$

where $D_0$ is the reference diffusion coefficient, and $\beta_T$ and $\beta_H$ are parameters that capture how the diffusion coefficient varies with temperature and humidity.

Sources or sinks of dust particles may be due to industrial emissions, vehicular pollution, biological processes, etc. For an industrial emission source with a relatively constant baseline, a simplified expression might be:

$$S(r,t,T,H) = S_0(r,t) \cdot f(T,H)$$

where $S_0(r,t)$ is the emission intensity at a given spatial location r and time t, and $f(T,H)$ is a weighting function representing how temperature and humidity conditions modify that intensity.

Combining the components described above, we obtain the complete model for the dust concentration C(r,t):

$$\frac{\partial C}{dt} + \nabla(u_0(1 + \alpha_T(T - T_0) + \alpha_H(H - H_0))C) = \nabla \cdot (D_0(1 + \beta_T(T - T_0) + \beta_H(H - H_0))\nabla C) + S_0(r,t) \cdot f(T,H)$$

To solve this partial differential equation (PDE), initial and boundary conditions must be specified. For instance:

**Initial condition:**

$$C(r,0) = C_0(r)$$

where $C_0(r)$ is the initial dust concentration distribution.

**Boundary conditions:**

- ✓ Dirichlet condition: $C(r_{\text{boundary}}, t) = C_{\text{boundary}}, (t),$
- ✓ Neumann condition: $(D\nabla C)|r_{boundary} = g(r_{\text{boundary}}, t),$
- ✓ or mixed conditions, depending on the physical setting.

For the problem to be well-posed:

1. The advection-diffusion equation is typically parabolic (and partly hyperbolic), and well-known existence and uniqueness theorems apply in suitable function spaces (e.g., Sobolev spaces).
2. If boundary and initial conditions are reasonably smooth, existence, uniqueness, and stability of the solution follow from standard theorems on linear PDEs.

**Outline of the proof:**

- ➢ Advection-diffusion equations can be addressed via Lax–Milgram theorem, semigroup methods, or Galerkin methods.
- ➢ Provided u(T,H) and D(T,H) are linear in T and H, and no nonlinearities in C (such as $C^2$), the equation remains linear in C. Therefore, under standard assumptions, existence and uniqueness of the solution are guaranteed.

Thus, this extended advection-diffusion equation (with linear dependence on temperature and humidity) can be solved using classical methods. The following sections demonstrate how to apply this model in practice and analyze the results. The simulation using the created model can be seen in the Matlab program. Figure 1.

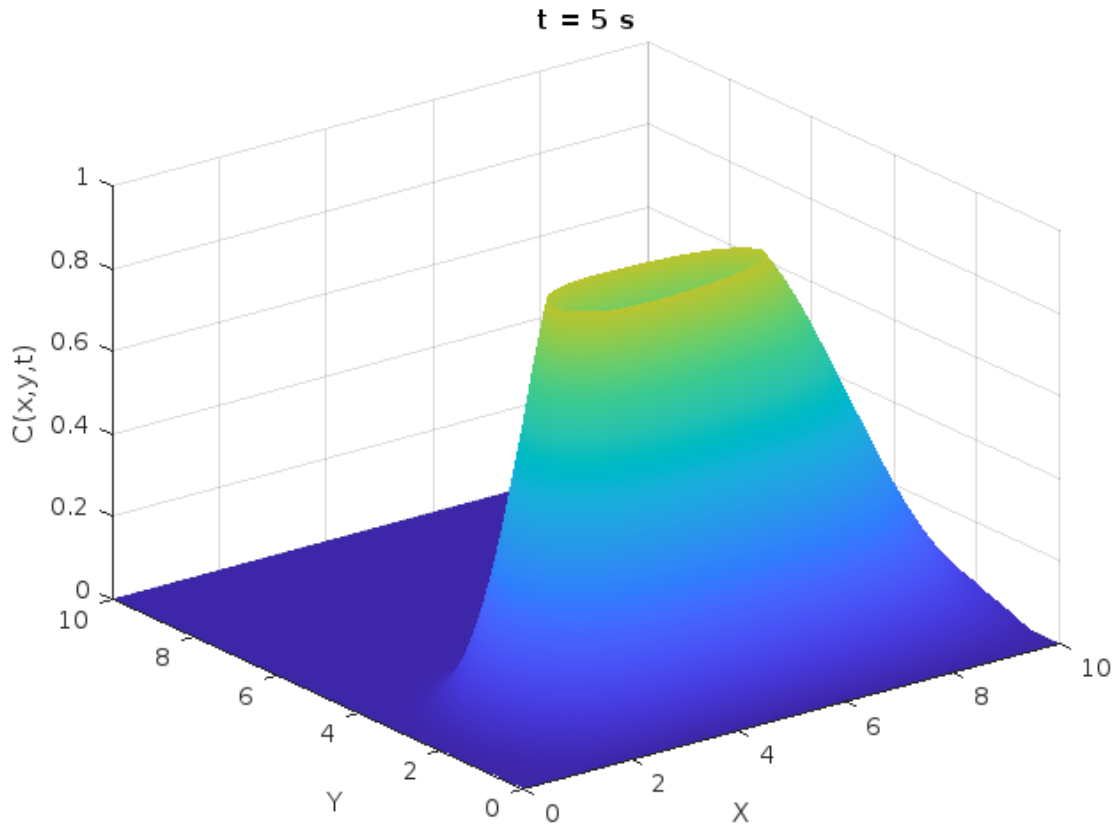


**Figure 1. Simulation results of the dispersion of dust particles in the air (depending on air temperature and humidity)**

### 3. Results

For computational simulation of dust dispersion, we specify the following parameters:

- **Spatial domain**: A 2D region (x,y) plane, $\Omega = \{(x,y)|0 \le x \le L_x,\ \ 0 \le y \le L_y\}$
- **Time interval:** $0 \le t \le T_{end}$.
- **Initial reference temperature** ($T_0$) and **initial reference humidity** ($H_0$): for instance, $T_0 = 20^0C \ and \ H_0 = 0.5$ (relative humidity).
- **Linear coefficients** $\alpha_T$=0.01, $\alpha_H$=0.02, $\beta_T$=0.005, $\beta_H$=0.004.
- **Diffusion coefficient** $D_0$=0.1 $m^2/s$.
- **Velocity** $u_0$=(1, 0) m/s (for example, a uniform flow in the horizontal direction).

Although simplified, these parameters illustrate the general trends in dust particle dispersion.

The computational results suggest:

1. **Effect of temperature increase** ($T>T_0$): If $\alpha_T>0$, an increase in temperature enhances the flow velocity, accelerating dust advection across the domain.

2. **Effect of increased humidity** ($H>H_0$): If $\beta_H>0$, rising humidity tends to reduce the diffusion coefficient, slowing down the rate at which dust spreads via diffusion.
3. **Source intensity** ($S_0(r,t)$): Higher emission intensities lead to higher peak concentrations, which then spread along the primary advection direction.

Plots of the spatial distribution of C(x,y,t) over time show more rapid dispersion in higher temperature areas and a slowdown in more humid regions. In extremely high humidity environments, particle agglomeration and sedimentation (gravitational settling) become more pronounced.

## 4. Discussion

From the results, several conclusions follow:

Impact of temperature: With increasing temperature, higher velocity can enhance dust transport through advection. Temperature also affects the system's overall energy, increasing the random motion of dust particles and thus influencing diffusion processes.

Impact of humidity: High humidity promotes particle aggregation, increasing their effective size and accelerating settling. Within the model, additional sink terms or nonlinear forms of βH\beta_HβH may be included to represent more realistic scenarios.

Limitations: The model relies on a linear approximation of temperature and humidity dependence. In real conditions, these relationships can be more complex. Moreover, physical and chemical properties (size, shape, and surface characteristics) of the dust particles themselves are simplified in this model.

Practical applications: Such a model can be used for preliminary assessments in designing ventilation systems, planning environmental monitoring in industrial areas or large cities, and evaluating measures for air pollution control. To enhance the reliability of predictions, empirical data should be used to refine and validate the model.

## 5. Conclusion

In this article, we investigated the dispersion of dust particles influenced by ambient temperature and humidity using a mathematical model. An advection-diffusion equation extended with linear temperature and humidity dependencies was proposed. Theoretical analyses show that the model is stable and provides useful insights into dust dispersion dynamics.

Simulation results confirm that higher temperatures can accelerate particle dispersion, while increased humidity can slow diffusion or induce particle agglomeration. Although the real environment may require more sophisticated, nonlinear models, the present formulation provides a foundational approach for environmental monitoring and protection.

Further developments would involve including nonlinear effects, accounting for different particle sizes and properties, and incorporating gravity and turbulence. Experimental validation remains crucial for calibrating the model parameters to real-world conditions.

Studying the dispersion of dust particles based on temperature and humidity is vital in many areas, including environmental monitoring, industrial safety, and public health. The mathematical model and code presented here describe a fundamental, simplified approach. Further research could incorporate nonlinearities, particle sedimentation, and turbulence. Experimental validation is crucial for accurately calibrating the model parameters and ensuring its relevance to real-world scenarios.

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