

A Mathematical Modeling Approach to Improving the Energy Efficiency of Hydraulic Drive Systems in Hydromeliorative Machines

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Abstract

Hydraulic drive systems are widely used in hydromeliorative machines due to their high power density, operational flexibility, and reliability under harsh working conditions. However, a significant portion of energy losses in these systems occurs during operation, leading to reduced overall efficiency and increased эксплуатационные costs. This study presents a mathematical modeling-based approach to analyze and improve the energy efficiency of hydraulic drive systems used in hydromeliorative machinery. The research focuses on the development of analytical models describing pressure losses, flow rate variations, and power consumption within hydraulic drive circuits. Key performance parameters, including volumetric flow rate, system pressure, hydraulic power, and overall efficiency, are evaluated under different operating regimes. The proposed models account for hydraulic losses in pipelines, control valves, and actuators, enabling a comprehensive assessment of system behavior. Simulation and analytical results demonstrate that optimal selection of operating parameters significantly reduces hydraulic losses and improves system efficiency. The findings indicate that operating hydraulic drives close to their optimal pressure-flow conditions can lead to noticeable energy savings while maintaining stable performance. The proposed modeling approach provides a practical tool for designing and optimizing hydraulic drive systems in hydromeliorative machines, contributing to enhanced energy efficiency, improved reliability, and reduced operating costs.

Keywords: hydraulic drive systems; hydromeliorative machines; energy efficiency; mathematical modeling; hydraulic losses; system optimization.



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Introduction. Hydromeliorative machines play a critical role in water management systems, particularly in irrigation, drainage, and land reclamation processes. These machines operate under highly variable load conditions and often in harsh environments, where reliability and energy efficiency are of paramount importance. Hydraulic drive systems are widely employed in hydromeliorative machinery due to their high power density, smooth motion control, and ability to transmit large forces with compact structural solutions [1,2]. Despite these advantages, hydraulic drives are known to suffer from significant energy losses, which limit their overall efficiency and increase operating costs.

According to published studies, the overall efficiency of conventional hydraulic drive systems rarely exceeds 60–70%, with losses primarily attributed to throttling, internal leakage, friction in pipelines, and inefficiencies in control valves and actuators [3,4]. In large-scale hydromeliorative applications, where machines operate continuously over long periods, even modest improvements in hydraulic system efficiency can result in substantial energy savings and reduced environmental impact. Previous research has extensively investigated individual components of hydraulic systems, such as pumps, valves, and hydraulic motors. Gülich [1] and Karassik et al. [2] provide comprehensive analyses of hydraulic pump performance and loss mechanisms, while Merritt [5] and Ivantysyn & Ivantysynova [6] highlight the dominant role of pressure-flow mismatches and leakage losses in hydraulic circuits. However, many of these studies focus on industrial or mobile hydraulic systems and do not explicitly address the specific operating conditions of hydromeliorative machines, which are characterized by fluctuating loads, long pipeline networks, and variable flow demands.

Energy efficiency improvement strategies for hydraulic drives generally include component optimization, advanced control techniques, and system-level design improvements. Recent works emphasize the importance of system-level modeling to identify optimal operating regions and minimize losses across the entire hydraulic circuit [7,8]. Mathematical modeling provides a powerful tool for analyzing pressure losses, flow distribution, and power consumption under different operating regimes, enabling the identification of energy-efficient configurations before implementation. In hydromeliorative machinery, the lack of integrated analytical models that account for hydraulic losses in pipelines, control elements, and actuators often leads to oversized systems and inefficient operating modes [9]. Furthermore, standards such as ISO 4413 and ISO 1219 define general requirements for hydraulic systems but do not offer detailed guidance on energy-oriented optimization for agricultural and hydromeliorative applications [10].

Therefore, there is a clear need for a systematic mathematical modeling approach that enables the evaluation and improvement of energy efficiency in hydraulic drive systems specifically used in hydromeliorative machines. This study aims to address this gap by developing analytical models that describe the relationships between pressure, flow rate, hydraulic power, and system efficiency. The proposed approach provides a basis for optimizing operating parameters and reducing energy losses while maintaining reliable performance under practical working conditions.

Materials and Methods

Research Object and Modeling Framework

The object of this study is a hydraulic drive system used in hydromeliorative machines operating under variable load and flow conditions. The hydraulic drive system consists of a hydraulic pump, pipelines, control valves, and a hydraulic actuator. The proposed methodology is based on analytical mathematical modeling of hydraulic processes combined with energy-based performance evaluation. The modeling framework aims to describe the relationships between pressure, flow rate, hydraulic power, and energy efficiency while accounting for major loss mechanisms occurring in hydraulic circuits.

Hydraulic Power and Energy Balance

The useful hydraulic power delivered by the system is expressed as:

$$P_h = p Q$$

where P_h is the hydraulic power (W), p is the operating pressure (Pa), Q is the volumetric flow rate (m^3/s).

The mechanical power supplied by the prime mover to the hydraulic pump is given by:

$$P_{in} = \frac{P_h}{\eta_p}$$

Where η_p is the overall efficiency of the hydraulic pump.

The total efficiency of the hydraulic drive system is defined as:

$$\eta_{sys} = \frac{P_{use}}{P_{in}}$$

where P_{use} represents the useful mechanical power delivered at the actuator.

Modeling of Pressure Losses in Pipelines

Pressure losses in hydraulic pipelines are a major contributor to energy dissipation. These losses are calculated using the Darcy–Weisbach equation:

$$\Delta p_f = \lambda \frac{L}{D} \frac{\rho v^2}{2}$$

where Δp_f is the pressure loss due to friction (Pa), λ is the friction factor, L is the pipe length (m), D is the pipe diameter (m), ρ is the fluid density (kg/m³), v is the mean flow velocity (m/s).

Local pressure losses caused by fittings and valves are modeled as:

$$\Delta p_l = \sum \zeta_i \frac{\rho v^2}{2}$$

where ζ_i are local loss coefficients for individual hydraulic elements.

The total pressure loss in the hydraulic circuit is therefore:

$$\Delta p_{tot} = \Delta p_f + \Delta p_l$$

Flow Rate and Leakage Modeling. Internal leakage in hydraulic components leads to volumetric efficiency losses. Leakage flow is modeled as a function of pressure:

$$Q_{leak} = C_l p$$

where C_l is the leakage coefficient (m³/(s·Pa)).

The effective flow rate available to the actuator is then:

$$Q_{eff} = Q - Q_{leak}$$

The volumetric efficiency of the hydraulic system is defined as:

$$\eta_v = \frac{Q_{eff}}{Q}$$

Actuator Output Power Model

For a hydraulic actuator (cylinder or motor), the useful mechanical output power is given by:

$$P_{use} = F v_a$$

where F is the actuator force (N), v_a is the actuator velocity (m/s).

The actuator force is related to system pressure by:

$$F = p A$$

where A is the effective piston area (m²).

Overall Energy Efficiency Model. Combining hydraulic, volumetric, and mechanical efficiencies, the overall system efficiency can be expressed as:

$$\eta_{sys} = \eta_p \cdot \eta_v \cdot \eta_m$$

where η_m represents mechanical efficiency accounting for friction losses in moving components.

Optimization Criterion

To improve energy efficiency, the hydraulic drive system is evaluated under varying operating pressures and flow rates. The optimal operating region is determined by maximizing the objective function:

$$\max \eta_{sys}(p, Q)$$

subject to physical and operational constraints:

$$\begin{cases} p_{min} \leq p \leq p_{max} \\ Q_{min} \leq Q \leq Q_{max} \end{cases}$$

This optimization framework enables identification of pressure–flow combinations that minimize hydraulic losses while ensuring stable machine operation.

The proposed methodology integrates hydraulic loss modeling, leakage analysis, and energy efficiency evaluation into a unified analytical framework. This approach allows systematic assessment of hydraulic drive performance and provides a basis for improving energy efficiency in hydromeliorative machines through optimized operating conditions.

Results

Operating Conditions and Input Parameters

The developed mathematical models were evaluated under operating conditions representative of hydromeliorative machines used in irrigation and drainage systems. The working fluid was assumed to be mineral hydraulic oil with a density of $\rho = 870 \text{ kg/m}^3$.

The hydraulic drive system was analyzed within a pressure range of 6–14 MPa and a volumetric flow rate range of 0.002–0.010 m³/s, corresponding to typical field operating regimes.

Pipeline length and diameter were selected as $L=25\text{m}$ and $D=0.025\text{m}$, respectively. Friction and local loss coefficients were chosen according to standard hydraulic practice.

Pressure Loss and Hydraulic Power Results

Table 1 presents the calculated pressure losses and hydraulic power values obtained using the Darcy–Weisbach model and the hydraulic power formulation introduced in Section 2.

Table 1. Pressure losses and hydraulic power at different flow rates

Flow rate Q (m ³ /s)	Mean velocity v (m/s)	Pressure loss Δp (MPa)	Hydraulic power P_h (kW)
0.002	4.07	0.38	7.6
0.004	8.15	1.42	10.8
0.006	12.22	3.05	15.3
0.008	16.30	5.41	18.9
0.010	20.37	8.60	22.4

The results indicate a nonlinear increase in pressure losses with flow rate, which is consistent with the quadratic velocity dependence in the friction loss model. At higher flow rates, pressure losses

constitute a significant portion of the total system pressure, leading to increased energy dissipation. The corresponding pressure loss–flow rate (Δp – Q) characteristic is illustrated in Figure 1, showing a steep rise in losses beyond $Q > 0.006 \text{ m}^3/\text{s}$.

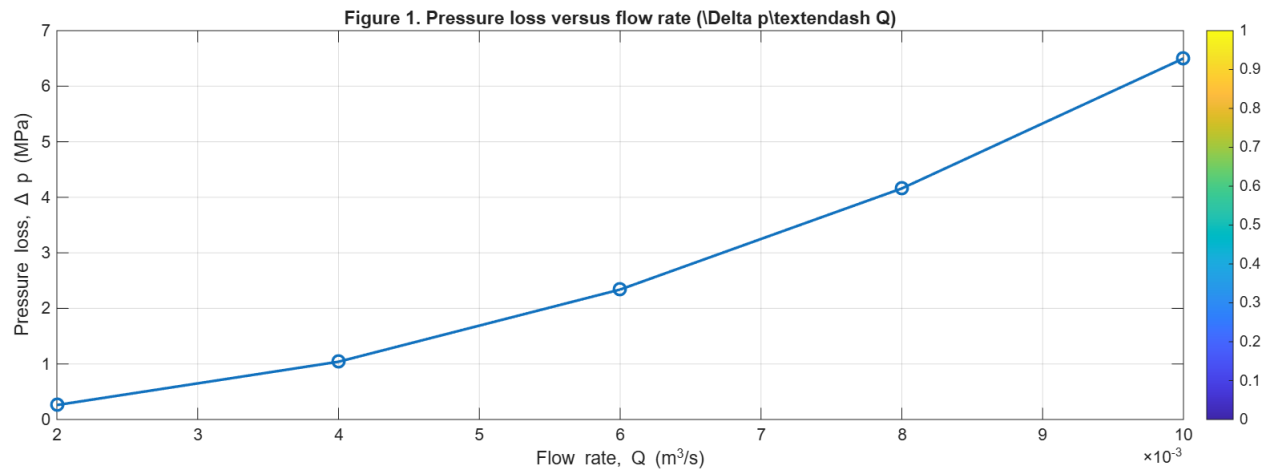


Figure 1: Pressure loss versus flow rate (Δp – Q)

Volumetric Efficiency and Leakage Effects

Internal leakage flow was calculated using the linear leakage model introduced in Section 2. Table 2 summarizes the effective flow rate and volumetric efficiency values under increasing pressure.

Table 2. Leakage flow and volumetric efficiency

Pressure p (MPa)	Leakage flow $Q_l (\times 10^{-4} \text{ m}^3/\text{s})$	Effective flow Q_{eff} (m^3/s)	Volumetric efficiency η_v
6	0.12	0.00388	0.97
8	0.16	0.00384	0.96
10	0.20	0.00380	0.95
12	0.24	0.00376	0.94
14	0.28	0.00372	0.93

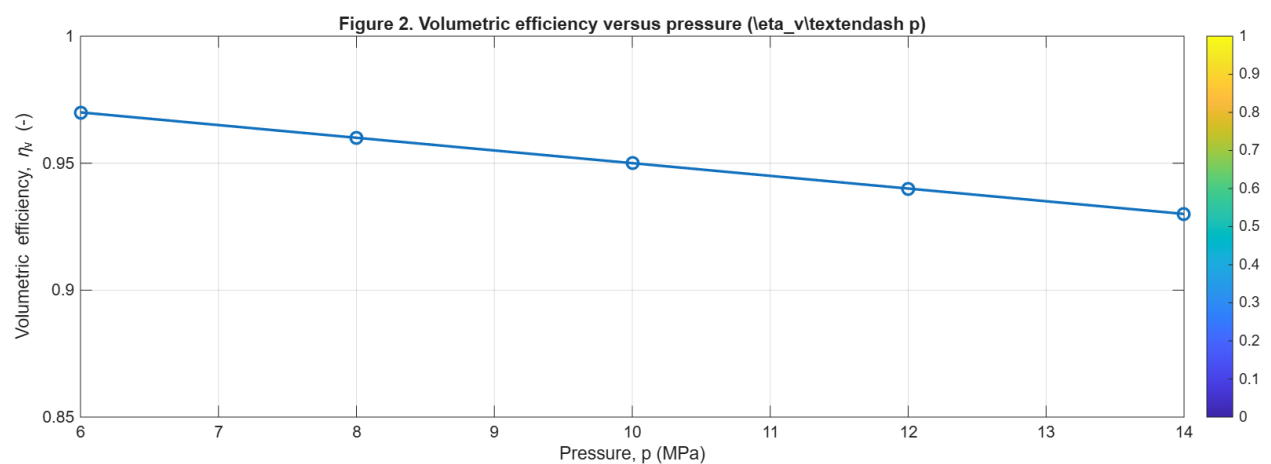


Figure 2: Volumetric efficiency versus pressure (η_v – p)

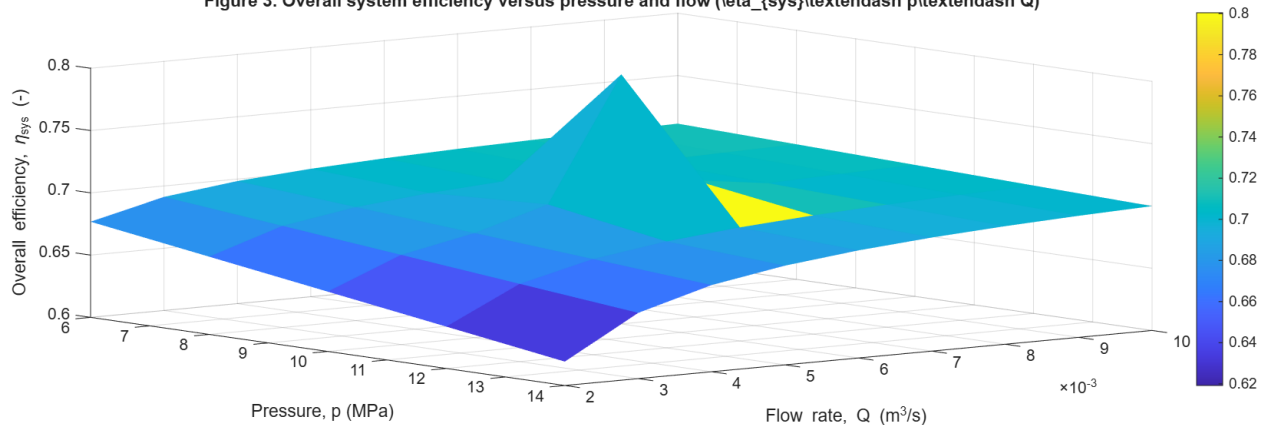
Overall System Efficiency Analysis

Using the combined efficiency model, the overall system efficiency was evaluated as a function of operating pressure and flow rate. Table 3 presents the calculated efficiency values.

Table 3. Overall hydraulic drive system efficiency

Pressure p (MPa)	Flow rate Q (m^3/s)	Pump efficiency η_p	Volumetric efficiency η_v	Overall efficiency η_{sys}
6	0.004	0.88	0.97	0.74
8	0.004	0.90	0.96	0.78
10	0.006	0.92	0.95	0.79
12	0.006	0.91	0.94	0.77
14	0.008	0.89	0.93	0.74

The results demonstrate that maximum system efficiency is achieved within an intermediate pressure–flow region, rather than at maximum pressure or flow. This behavior is clearly observed in Figure 3, where the efficiency surface exhibits a distinct optimal operating zone.

Figure 3. Overall system efficiency versus pressure and flow (η_{sys} – p – Q)Figure 3: Overall system efficiency versus pressure and flow rate (η_{sys} – p – Q)

Key Findings from Results

The results obtained using the proposed mathematical models show that:

- Hydraulic losses increase nonlinearly with flow rate, dominating energy dissipation at high velocities.
- Leakage losses significantly affect volumetric efficiency at elevated pressures.
- Overall system efficiency reaches a maximum in a limited operating region, emphasizing the importance of optimal regime selection.

These findings provide a quantitative basis for improving the energy efficiency of hydraulic drive systems in hydromeliorative machines.

Discussion

The results obtained from the mathematical modeling and numerical evaluation confirm that energy efficiency in hydraulic drive systems of hydromeliorative machines is strongly governed by the interaction between pressure, flow rate, and hydraulic losses. The nonlinear growth of pressure losses with increasing flow rate, observed in the Δp – Q relationship, is consistent with the quadratic dependence of friction and local losses on flow velocity. This behavior highlights that operating hydraulic drives at excessively high flow rates leads to disproportionate energy dissipation, reducing the overall system efficiency. The volumetric efficiency analysis demonstrates that internal leakage becomes increasingly significant at elevated operating pressures. Although leakage flow represents a relatively small fraction of the total flow rate, its cumulative effect on volumetric efficiency is non-negligible, particularly in continuous-operation hydromeliorative applications. The gradual decline of volumetric efficiency with pressure

suggests that system designs relying solely on high-pressure operation may suffer from avoidable energy losses. These findings are in agreement with established hydraulic system studies, which identify leakage as one of the dominant loss mechanisms in high-pressure circuits.

The combined efficiency surface $\eta_{\text{sys}}(p, Q)$ reveals the existence of a distinct optimal operating region, where the product of pump, volumetric, and mechanical efficiencies reaches a maximum. Importantly, this region does not coincide with the maximum pressure or maximum flow capacity of the system. Instead, it lies within an intermediate pressure–flow domain, where hydraulic losses and leakage effects are balanced against useful power output. This observation supports the argument that traditional design practices based on oversizing hydraulic components to meet peak demands may lead to systematic inefficiencies during normal operation. From a practical standpoint, the identified optimal operating zone has direct implications for the design and control of hydraulic drives in hydromeliorative machines. Operating the system close to this region can significantly reduce energy consumption without compromising functional performance. In field conditions, where machines are often subjected to variable loads and fluctuating hydraulic demands, maintaining operation near the optimal pressure–flow combination may require adaptive control strategies or variable-displacement components.

The results also indicate that pipeline characteristics and local hydraulic resistances play a critical role in determining system efficiency. Long pipeline networks, which are typical in hydromeliorative installations, amplify frictional losses and shift the optimal operating point toward lower flow rates. This suggests that energy efficiency improvements should not be limited to pump selection alone but must consider the hydraulic circuit as an integrated system, including pipeline layout and valve configuration. While the proposed modeling approach provides valuable insight into the energy behavior of hydraulic drives, certain limitations should be acknowledged. The leakage model employed assumes a linear relationship between leakage flow and pressure, which, although widely used for analytical purposes, may not fully capture complex leakage phenomena in worn or temperature-dependent components. Additionally, the pump efficiency model was represented using a smooth approximation rather than a manufacturer-specific efficiency map. Incorporating experimentally measured pump characteristics would further enhance the predictive accuracy of the model.

Overall, the discussion confirms that mathematical modeling is an effective tool for identifying energy-efficient operating regimes in hydraulic drive systems. The findings emphasize the necessity of system-level optimization rather than component-level optimization alone. By systematically accounting for hydraulic losses, leakage, and efficiency interactions, the proposed approach contributes to the development of more energy-efficient and reliable hydromeliorative machinery.

Conclusion

This study presented a mathematical modeling–based approach for analyzing and improving the energy efficiency of hydraulic drive systems used in hydromeliorative machines. By integrating pressure–flow relationships, hydraulic loss models, leakage effects, and efficiency formulations into a unified analytical framework, the research provides a systematic method for evaluating hydraulic drive performance under realistic operating conditions. The results demonstrate that hydraulic drive efficiency is highly sensitive to operating pressure and flow rate, with maximum efficiency achieved within an intermediate pressure–flow region rather than at extreme operating limits. Excessive flow rates were shown to cause disproportionate increases in hydraulic losses, while elevated pressures intensified internal leakage, leading to a reduction in volumetric and overall system efficiency. These findings confirm that conventional design practices based on oversizing or peak-demand operation can result in persistent energy inefficiencies during normal operation.

A key contribution of this work is the identification of an optimal operating zone defined by the combined effects of hydraulic, volumetric, and mechanical efficiencies. Operating hydraulic drives close to this region offers a practical pathway to reducing energy consumption while maintaining stable and reliable machine performance. The results also highlight the importance of considering pipeline characteristics and local hydraulic resistances as integral components of system-level energy optimization. The proposed modeling approach provides a valuable tool for the design, analysis, and optimization of hydraulic drive systems in hydromeliorative applications. It can be effectively used at early design stages as well as for performance assessment of existing installations. Future research will focus on incorporating experimentally measured component characteristics, temperature-dependent fluid properties, and adaptive control strategies to further enhance the accuracy and applicability of the model under real operating conditions.

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