E-ISSN: 2997-9382
American Journal of Technology Advancement
https://semantjournals.org/index.php/AJTA

| Research Article

Check for updates

Predicting Ion Transport Properties of Electrolytes Via Regression Modeling and Assessment of Factor Influence

A. Qosimov, A. Tursunov, J. Shodmanov
Namangan State Technical University

Abstract
In electrochemical systems, ion transport and charge transfer kinetics at the electrode–electrolyte interface are fundamental processes that dictate overall device efficiency. In this study, the dependence of ionic conductivity sigma (S·cm⁻¹) and charge transfer resistance R_{ct} (Omega·cm²) on temperature (T), KOH concentration (c), and mechanical deformation (varepsilon) was modeled using regression analysis based on a central non-composite design. Second-order multi-factor regression equations were constructed, with coefficient significance evaluated via Student's t-test and model adequacy verified through the Fisher criterion. Consequently, a simplified equation for sigma (comprising only significant terms) was derived, and conclusions regarding dominant factor influence directions and optimization were formulated.

Keywords: ionic conductivity, charge transfer resistance, regression model, Student's criterion, Fisher criterion, KOH electrolyte, deformation.

This is an open-access article under the CC-BY 4.0 license

Introduction

In electrochemical energy systems (such as supercapacitors, batteries, and electrochemical sensors), two critical indicators—**ionic conductivity** (sigma) and **charge transfer resistance** (R_{ct})—directly influence operational speed, internal losses, power density, and stability. While ionic conductivity represents the movement of ions within the electrolyte or membrane bulk, R_{ct} is regarded as the kinetic "barrier" to electron-ion exchange at the electrode–electrolyte interface. Therefore, expressing the dependence of sigma and R_{ct} on technological factors through mathematical modeling is of high practical importance for reducing experimental volume and identifying optimal operating regimes.

The fundamental processes occurring in these systems are decisive factors for overall efficiency. R_{ct} (Omega·cm²) characterizes the kinetic barriers of the exchange process at the phase boundary. Analyzing the variations of these two indicators and determining their relationship with input technological factors is essential for establishing optimal system performance conditions.

Methodology and Factor Selection

During the research, temperature (T), KOH concentration (c), and mechanical deformation (varepsilon) were selected as the primary physicochemical factors. These parameters directly affect internal ion transport processes, reaction rates at the interface, and the structural state of the material.

To construct the model, a **full quadratic regression equation** was selected based on a **Central Composite Design (CCD)**. This approach allows for the identification of linear, quadratic, and interaction effects between factors. The objective of the regression model is to predict ionic conductivity and charge transfer resistance through mathematical expressions based on specific combinations of input factors.

The input variables are defined as follows:

- ✓ **X_1**: Temperature (T, ^circtext{C})
- ✓ **X_2**: KOH Concentration (c, text{M})
- ✓ **X_3**: Deformation (varepsilon)

Factor Analysis

1. **Temperature (T, ^circtext{C}):** As a primary energy activation factor, increasing temperature enhances ionic mobility and reduces internal resistance, thereby increasing sigma. Simultaneously, higher temperatures accelerate kinetic reactions at the interface, reducing R_{ct}. The study utilized a range of **10°C to 60°C**, ensuring optimal experimental conditions relative to the material's thermal stability.
2. **KOH Concentration (c, text{M}):** This is directly linked to the quantity and mobility of ions. While increasing KOH concentration initially improves conductivity by increasing ion density, excessive concentrations may trigger ion association, leading to a decline in conductivity. This factor was investigated within the **2–12 M** range.
3. **Deformation (varepsilon):** Mechanical strain alters the structure of ion channels under tension or pressure, affecting the geometry of transport pathways and the active surface area of the electrode. While minor deformations may enhance transport, excessive strain disrupts the network structure, decreasing sigma and increasing R_{ct}. The range was set between **0 and 0.5 (0–50%)** to remain within the material's mechanical limits.

Table 1 outlines the levels and intervals of variation for the factors under investigation.

Table 1: Selection of variation levels and intervals for the investigated factors.

Name and designation of factors	Change levels			Change interval
	-1	0	+1	
x_1- Temperature, T (℃)	10	35	60	25
x_2- KOH concentration, c (m)	0	1	2	1
x_3- Deformation, ε	0	0,25	0,5	0,25

In determining the regression coefficients, Student's t-test was employed, while the Fisher (F-test) criterion was utilized to verify the adequacy of the mathematical model. The response variables (output factors) were defined as Y_1: Ionic conductivity (sigma, $S \cdot cm^{-1}$) and Y_2: Charge transfer resistance (R_{ct}, Omega·cm^2).

The process under investigation is characterized by higher-order complexities. Consequently, to derive a second-order mathematical regression model, the Central Non-Composite Design

(CNCD) was selected and implemented. This method was chosen for its relative simplicity and efficiency compared to alternative approaches, as well as its proven efficacy in modeling complex technological processes within industrial research.

2-table. Central non-compositional experience matrix

№	Factors			x_1x_2	x_1x_3	x_2x_3	x_1^2	x_2^2	x_3^2	Y_1	Y_2	$S_u^2(Y_1)$	$S_u^2(Y_2)$
	x_1	x_2	x_3										
1	+	+	0	+	0	0	+	+	0	4,112	-5,755	0,131	0,0032
2	+	-	0	-	0	0	+	+	0	4,112	-5,155	0,072	0,0058
3	-	+	0	-	0	0	+	+	0	3,212	-4,955	0,059	0,0070
4	-	-	0	+	0	0	+	+	0	3,912	-3,555	0,022	0,0286
5	+	0	+	0	+	0	+	0	+	6,262	-5,155	0,076	0,0058
6	+	0	-	0	-	0	+	0	+	4,962	-5,655	0,125	0,0035
7	-	0	+	0	-	0	+	0	+	5,282	-3,855	0,025	0,0212
8	-	0	-	0	+	0	+	0	+	3,802	-4,555	0,051	0,0105
9	0	+	+	0	0	+	0	+	+	4,462	-4,905	0,059	0,0074
10	0	+	-	0	0	-	0	+	+	4,862	-5,605	0,119	0,0037
11	0	-	+	0	0	-	0	+	+	3,562	-4,005	0,029	0,0182
12	0	-	-	0	0	+	0	+	+	3,162	-4,505	0,048	0,0111
13	0	0	0	0	0	0	0	0	0	4,115	-4,505	0,002	0,0100
14	0	0	0	0	0	0	0	0	0	4,325	-4,521	0,009	0,0100
15	0	0	0	0	0	0	0	0	0	4,212	-4,421	0,051	0,010002
The values (+), (-) and 0 must be used in the matrix.													

Based on the experimental results, we search for a second-order regression multifactor mathematical model. As a result of this experiment, we can obtain a regression model with the following general form:

$$Y_R = b_0 + \sum_{i=1}^{M} b_i x_i + \sum_{\substack{i=j=1 \\ j\neq 1}}^{n} b_{ij} x_i x_j + \sum_{i=1}^{M} b_{ii} x_i^2$$

Or, since there are three factors involved in our experiment, the above expression takes the form:

$$Y_R = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2$$

In the equation: b_0…b_1… - regression coefficients, x_1,x_2,x_3 - coded values of factors.

$g_1 = 0{,}2$ $g_2 = 0{,}17$ $g_5 = 0{,}13$ $g_3 = 0{,}13$

$g_6 = 0{,}06$ $g_4 = 0$ $g_7 = 0{,}31$

Y1- Calculation of models for ionic conductivity optimization and determination of regression coefficients:

$$b_0 = \frac{1}{N_s}\sum_{u=1}^{N_s} \overline{Y}_{\overline{u}} = \frac{1}{3}(4{,}115 + 4{,}325 + 4{,}212) = 4{,}22$$

$$b_i = g_3 \sum_{u=1}^{N} x_{iu}\overline{Y}_u$$

b_1 =0,125(4,11+4,41+(-3,21)+(-3,91)+6,26+4,96+(-5,28)+(-3,8)=0,440

b_2 =0,125(4,11+(-4,41)+ 3,21+(-3,91)+4,46+4,86+(-3,56)+(-3,16)= 0,200

b_3 =0,125(6,26+(-4,96)+5,28+(-3,8)+4,46+(-4,86)+3,56+(-3,16)= 0,350

$$b_{ij} = g_4 \sum_{u=1}^{N} x_{iu} x_{ju} \overline{Y}_u$$

b_{12} =0,25(4,11+(-4,41)+(-3,21)+3,91)= 0,100

b_{13} =0,25(6,25+(-4,96)+(-5,28)+3,8)=-0,050

b_{23} =0,25(4,46+(-4,86)+(-3,56)+3,16)= -0,200

$$b_{ii} = g_5 \sum_{u=1}^{N} x_{iu}^2 \overline{Y}_u + g_6 \sum_{i=1}^{M} \sum_{u=1}^{N} x_{iu}^2 \overline{Y}_u - g_2 \sum_{u=1}^{N} \overline{Y}_u$$

$$\sum x_1^2 \overline{Y_u} = 4{,}11 + 4{,}41 + 3{,}21 + 3{,}91 + 6{,}26 + 4{,}96 + 5{,}28 + 3{,}8 = 35{,}940$$

$$\sum x_2^2 \overline{Y_u} = 4{,}11 + 4{,}41 + 3{,}21 + 3{,}91 + 4{,}46 + 4{,}86 + 3{,}56 + 3{,}16 = 31{,}680$$

$$\sum x_3^2 \overline{Y_u} = 6{,}26 + 4{,}96 + 5{,}28 + 3{,}8 + 4{,}46 + 4{,}86 + 3{,}56 + 3{,}16 = 36{,}340$$

$$\sum \overline{Y_u} = 4{,}22 + 4{,}412 + 3{,}212 + 3{,}912 + 6{,}212 + 4{,}964 + 5{,}282, +3{,}802 + 4{,}462 + 4{,}862 + 3{,}562 + 3{,}162 + 4{,}115 + 4{,}325, +4{,}212 = 64{,}66$$

$$\sum_{i=1}^{M} \sum x_i^2 \overline{Y_u} = 35{,}94 + 31{,}68 + 36{,}34 = 103{,}96$$

b_{11}= 0,125*35,94+ 0,0625*103,96+ 0,166*64,656 =0,26

b_{22}= 0,125*31,68 +0,0625*103,96+ 0,166*64,656 =-0,28

b_{33}= 0,125*36,34+ 0,0625*103,96+ 0,166*64,656 =0,31

Taking into account the determined regression coefficients, the equation is written:

$$Y_{R1} = 4{,}22 + 0{,}44x_1 + 0{,}2x_2 + 0{,}35x_3 + 0{,}1x_1x_2 - 0{,}05x_1x_3 - 0{,}2x_2x_3 + 0{,}26x_1^2 - 0{,}28x_2^2 + 0{,}31x_3^2$$

Y1- Determining the significance of the regression coefficients for optimizing ion permeability, for this purpose, the variance of the output parameter is determined and, on this basis, the variance in determining the regression coefficients is calculated:

$$S^2\{Y\} = S_m^2\{Y\} = \frac{1}{N_s - 1} \sum_{u=1}^{N_s} S^2\{\overline{Y}\}$$

$S^2\{\overline{Y}\} = \frac{1}{3-1} \cdot 0{,}062 = 0{,}0309$

$S^2\{b_0\} = g_1 S^2\{\overline{Y}\}$ =0,2∗0,0309= 0,006

$S^2\{b_i\} = g_3 S^2\{\overline{Y}\}$ =0,125∗0,0309=0,0039

$S^2\{b_{ij}\} = g_4 S^2\{\overline{Y}\} = 0{,}25 \cdot 0{,}0309$ =0,0077

$S^2\{b_{ii}\} = g_7 S^2\{\overline{Y}\} = 0{,}3125 \cdot 0{,}0309$ =0,0097

The mean square deviation in determining the regression coefficients is found:

$S\{b_0\}$ =0,0786 ; $S\{b_i\}$ = 0,0621 ; $S\{b_{ij}\}$ =0,0879 ; $S\{b_{ii}\}$ =0,0983

Then, the calculated value of the Student's test is determined using the following equation:

$$t_R\{b_i\} = \frac{|b_i|}{S\{b_i\}}$$

$$t_R\{b_0\} = \frac{|4,22|}{0,0786} = 53,69 \qquad t_R\{b_{12}\} = \frac{|0,1|}{0,0879} = 1,14$$

$$t_R\{b_1\} = \frac{|0,44|}{0,0621} = 5,69 \qquad t_R\{b_{13}\} = \frac{|0,05|}{0,0879} = 0,57$$

$$t_R\{b_2\} = \frac{|0,2|}{0,0621} = 3,22 \qquad t_R\{b_{23}\} = \frac{|0,2|}{0,0879} = 2,28$$

$$t_R\{b_3\} = \frac{|0,35|}{0,0621} = 5,64 \qquad t_R\{b_{11}\} = \frac{|0,26|}{0,0983} = 2,64$$

$$t_R\{b_{22}\} = \frac{|0,28|}{0,0983} = 2,85 \qquad t_R\{b_{33}\} = \frac{|0,31|}{0,0983} = 3,15$$

The tabular value of the Student criterion is taken from Appendix 3:

$t_j[P_D = 0,95; f\{S_s^2\} = 3 - 1 = 2] = 2,77$

It is known that if the calculated value of the criterion is less than the table value, then that coefficient is not significant and is removed from the equation. In the studies, it was found that the coefficient b_12, b_13, b_23 b_11 is insignificant for the parameters under study:

The equation is rewritten with significant coefficients:

$$Y_{R1} = 4,22 + 0,44x_1 + 0,2x_2 + 0,35x_3 - 0,28x_2^2 + 0,31x_3^2$$

Checking the obtained Y1- Ion conductivity equations for adequacy. The check is carried out using the Fisher criterion. The calculated value of the Fisher criterion is determined. The calculated value of the factor being optimized is calculated by substituting the coded values of all columns of the matrix (-1, 0 and +1) of the Y1 equation in Table 3. The values are obtained row by row, not column by column. The calculations for the Y formula are as follows, and the calculation results are included in Table 4:

In order to check whether the above regression mathematical model is adequate or not, we determine using the calculated value of the Fisher criterion.

$$F_R = \frac{S_{nad}^2\{Y\}}{S^2\{\overline{Y}\}}$$

where:

$$S^2\{\overline{Y_1}\} = \frac{\sum_{u=1}^{N} S^2\{Y\}}{N_s - 1} \qquad S_{nad}^2\{Y\} = \frac{\sum_{u=1}^{N-N_s+1} (Y_{Ru} - \overline{Y}_u)^2}{N - N_{k.en} - (N_s - 1)^2};$$

$$N - N_{k.en} - (N_s - 1)^2 = 15 - 6 - (3 - 1)^2 = 5$$

$$N - N_s + 1 = 15 - 3 + 1 = 13$$

Calculate the resulting Y1-Ion conductivity by substituting the encoded values into the equation:

$$S^2\{Y_1\} = \frac{\sum_{u=1}^{N} s^2\{Y\}}{N_s - 1} = \frac{28{,}70}{2} = 14{,}35$$

$$Y_{R1} = 4{,}22 + 0{,}44x_1 + 0{,}2x_2 + 0{,}35x_3 - 0{,}28x_2^2 + 0{,}31x_3^2$$

$Y_{R1.1}$ =4,22+0,44+0,2+(-0,28)=4,58

$Y_{R1.2}$ =4,22+0,44+(-0,2)+(-0,28)=4,18

$Y_{R1.3}$ =4,22+(-0,44)+0,2+(-0,28)=3,70

$Y_{R1.4}$ =4,22+(-0,44)+(-0,2)+(-0,28)=3,30

$Y_{R1.5}$ =4,22+0,44+0,35+0,31=5,28

$Y_{R1.6}$ =4,22+0,44+(-0,35)+0,31=4,62

$Y_{R1.7}$ =4,22+(-0,44)+0,35+0,31=4,94

$Y_{R1.8}$ = 4,22+(-0,44)+(-0,35)+0,31=3,74

$Y_{R1.9}$ =4,22+0,2+0,35+(-0,28)+0,31=4,80

$Y_{R1.10}$ =4,22+0,2+(-0,35)+(-0,28)+0,31=4,80

$Y_{R1.11}$ =4,22+(-0,2)+0,35+(-0,28)+0,31=3,70

$Y_{R1.12}$ =4,22+(-0,2)+(-0,35)+(-0,28)+0,31=3,70

3- table. The calculation results of the values encoded in the equation for adequate dispersion

No	Y_1-Ionic conductivity (σ (S·cm^{-1}))			
	Y_{1i}	Y_{1i}	$(Y_{1i}-Y_{R1i})$	$(Y_{1i}-Y_{R1i})^2$
1	4,112	4,58	0,47	0,22
2	4,412	4,18	-0,23	0,05
3	3,212	3,7	0,49	0,24
4	3,912	3,3	-0,61	0,37
5	6,262	5,82	-0,44	0,20
6	4,962	4,62	-0,34	0,12
7	5,282	4,94	-0,34	0,12
8	3,802	3,74	-0,06	0,00
9	4,462	4,8	0,34	0,11
10	4,862	4,8	-0,06	0,00
11	3,562	3,7	0,14	0,02
12	3,162	3,7	0,54	0,29

$$\sum_{u=1}^{N-N_s+1} (Y_{R1.u} - \overline{Y}_{1u})^2 = 1{,}75$$

$$S^2_{nad}\{Y_1\} = \frac{1{,}75}{4} = 0{,}44$$

It is known that if the calculated value of the criterion is less than the table value, then that coefficient is adequate and proves that the calculations were carried out correctly.

$$F_{R1} = \frac{S^2_{nad}\{Y\}}{S^2\{\overline{Y}\}} = \frac{0,44}{0,03088} = 14,25$$

$$F_j\left[P_D = 0,95;\ f\left\{S^2_{nad}\{Y\}\right\} = 15 - 6 - (3-1) = 5; f\{S^2_u\} = 3 - 1 = 2\right] = 4,74$$

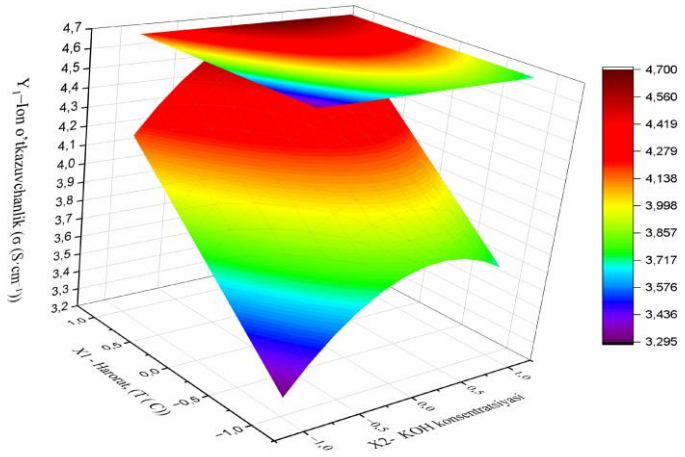
$$F_{R1} = 14,25 < 19,4 = F_j$$

Therefore, the obtained regression mathematical models represent the studied process with sufficient accuracy.

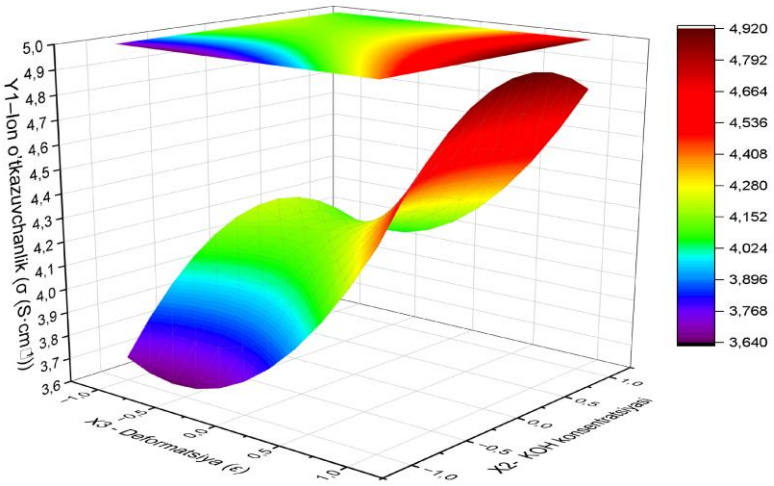
Obtaining optimization solutions and conclusions

Since the equation constructed to determine the characteristics of the output parameter for the study is three-dimensional, one of the input factors in the analysis is assumed to be X_i=0 (central position), and we construct a two-dimensional graph by transforming the models into 3 equations.

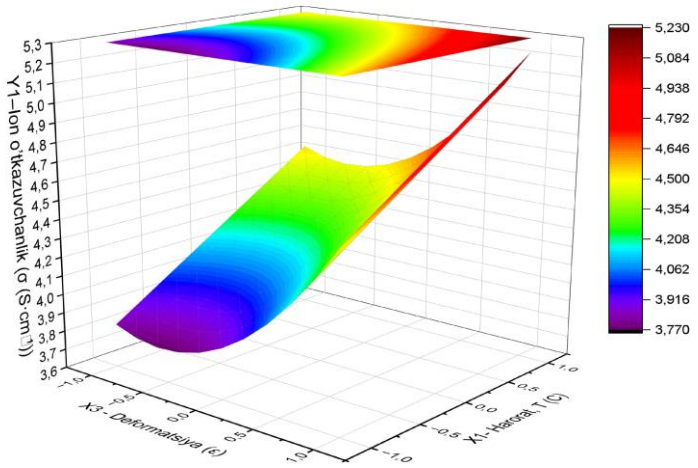
$$Y_{R1} = 4,22 + 0,44x_1 + 0,2x_2 + 0,35x_3 - 0,28x_2^2 + 0,31x_3^2$$



a-figure.



b-figure.



c-figure

Conclusion:

In this work, the dependence of ionic conductivity (σ) and charge transfer resistance (Rct) at the electrode–electrolyte interface on temperature (T), KOH concentration (c), and mechanical deformation (ε) was evaluated using second-order regression models based on central non-compositional design (CCD/MNKT). The significance of the regression coefficients was analyzed using the Student test, and the adequacy of the model was checked using the Fisher test. According to the results, it was observed that for σ, the temperature and deformation factors had a positive effect, while the KOH concentration had a nonlinear (quadratic) effect; this indicates the need for a balance between factors in determining the optimal operating modes. For Rct, it was hypothesized that an increase in temperature and concentration would reduce the resistance due to the acceleration of the interface kinetics, while deformation would increase the resistance after a certain threshold due to contact/structural deformation. The proposed regression models serve as a practical mathematical tool for predicting σ and Rct for combinations of technological factors and optimizing the energy efficiency of an electrochemical system..

Figure 1. Graph of the ionic conductivity optimization model.

Calculation of charge transfer resistance optimization models and determination of regression coefficients:

$$b_0 = \frac{1}{N_s}\sum_{u=1}^{N_s}\overline{Y_{\overline{u}}} = \frac{1}{3}(-4{,}505 + 4{,}5205 + 4{,}4205) = -4{,}48$$

b_1 =0,125((-5,76)+(-5,16)+4,96+3,56+(-5,16)+(-5,66)+3,86+4,56)= -0,60

b_2 =0,125(((-5,76)+5,16+(-4,96)+3,56+(-4,91)+(-5,61)+4,01+4,51)= -0,50

b_3 =0,125((-5,16)+5,66+(-3,86)+4,56+(-4,91)+5,61+(-4,01)+4,51)= 0,30

b_{12} =0,25((-5,76)+5,16+4,96+(-3,56))= 0,20

b_{13} =0,25((-5,16)+5,66+3,86+(-4,56))= -0,05

b_{23} =0,25((-4,91)+5,61+4,01+(-4,51))= 0,05

$$\sum x_1^2\overline{Y_u} = -5{,}76 + (-5{,}16) + (-4{,}96) + (-3{,}56) + (-5{,}16) + (-5{,}66) + (-3{,}86) + (-4{,}56) = -38{,}68$$

$$\sum x_2^2\overline{Y_u} = -5{,}76 + (-5{,}16) + (-4{,}96) + (-3{,}56) + (-4{,}91) + (-5{,}61) + (-4{,}01) + (-4{,}51) = -38{,}48$$

$$\sum x_3^2\overline{Y_u} = -5{,}16 + (-5{,}66) + (-3{,}86) + (-4{,}56) + (-4{,}91) + (-5{,}61) + (-4{,}01) + (-4{,}51) = -38{,}28$$

$$\sum \overline{Y_u} = -5{,}755 + (-5{,}155) + (-4{,}955) + (-3{,}555) + (-5{,}155) + (-5{,}655) + (-3{,}855) + (-4{,}555) + (-4{,}905) + (-5{,}605) + (-4{,}005) + (-4{,}505) + (-4{,}505) + (-4{,}5205) + (-4{,}4205) = -71{,}11$$

$$\sum_{i=1}^{M}\sum x_i^2\overline{Y_u} = -38{,}68 + (-38{,}48) + (-38{,}28) = -115{,}44$$

b_{11}=0,125*(-38,68)+0,0625*(-115,44)-0,166*$(-71{,}106)$=-0,25

b_{22}=0,125*$(-38{,}48)$+0,0625*(115,44)- 0,166*$(-71{,}106)$=-0,22

b_{33}=0,125*$(-38{,}28)$+ 0,0625*(-115,44)-0,166*$(-71{,}106)$=-0,20

Taking into account the determined regression coefficients, the equation is written:

$$Y_{R1} = -4{,}48 + (-0{,}6)x_1 + (-0{,}5)x_2 + 0{,}3x_3 + 0{,}2x_1x_2 + (-0{,}05)x_1x_3 + 0{,}05x_2x_3 + (-0{,}25)x_1^2 + (-0{,}22)x_2^2 + (-0{,}2)x_3^2$$

Y2- Determine the significance of the regression coefficients for charge transfer resistance optimization, for which the variance of the output parameter is determined and, based on this, the variance in determining the regression coefficients is calculated:

$$S^2\{Y\} = S_m^2\{Y\} = \frac{1}{N_s - 1}\sum_{u=1}^{N_s} S^2\{\overline{Y}\}$$

$$S^2\{\overline{Y}\} = \frac{1}{3-1}\cdot 0{,}030 = 0{,}0150$$

$$S^2\{b_0\} = g_1 S^2\{\overline{Y}\} = 0{,}2 \cdot 0{,}015 = \ 0{,}003$$

$$S^2\{b_i\} = g_3 S^2\{\overline{Y}\} = 0{,}125 \cdot 0{,}015 = 0{,}002$$

$$S^2\{b_{ij}\} = g_4 S^2\{\overline{Y}\} = 0{,}25 \cdot 0{,}015 = 0{,}004$$

$$S^2\{b_{ii}\} = g_7 S^2\{\overline{Y}\} = 0{,}3125 \cdot 0{,}015 = 0{,}005$$

The mean square deviation in determining the regression coefficients is found:

$$S\{b_0\} = 0{,}06; \quad S\{b_i\} = 0{,}043; \quad S\{b_{ij}\} = 0{,}061; \quad S\{b_{ii}\} = 0{,}07$$

Then, the calculated value of the Student's test is determined using the following equation:

$$t_R\{b_i\} = \frac{|b_i|}{S\{b_i\}}$$

$$t_R\{b_0\} = \frac{|-4{,}48|}{0{,}055} = 81{,}45 \qquad t_R\{b_{12}\} = \frac{|0{,}2|}{0{,}061} = 3{,}28$$

$$t_R\{b_1\} = \frac{|-0,6|}{0,043} = 13,95 \qquad t_R\{b_{13}\} = \frac{|-0,05|}{0,061} = 0,82$$

$$t_R\{b_2\} = \frac{|-0,5|}{0,043} = 11,63 \qquad t_R\{b_{23}\} = \frac{|0,05|}{0,061} = 0,82$$

$$t_R\{b_3\} = \frac{|0,3|}{0,043} = 6,98 \qquad t_R\{b_{11}\} = \frac{|-0,25|}{0,068} = 3,68$$

$$t_R\{b_{22}\} = \frac{|-2,07|}{1,71} = 1,21 \qquad t_R\{b_{33}\} = \frac{|-0,2|}{0,068} = 2,94$$

The tabular value of the Student criterion is taken from Appendix 3:

$$t_j[P_D = 0,95; f\{S_s^2\} = 3 - 1 = 2] = 2,77$$

It is known that if the calculated value of the criterion is less than the table value, then this coefficient is not significant and is removed from the equation. In the studies, it was found that the coefficients b_13, (b_23,)_(,) are insignificant for the parameters under study:

The equation is rewritten with significant coefficients:

$$Y_{R1} = -4,48 - 0,6x_1 - 0,5x_2 + 0,3x_3 + 0,2x_1x_2 - 0,25x_1^2 - 0,22x_2^2 - 0,2x_3^2$$

Check the obtained Y2- Charge transfer resistance equations for adequacy. The check is carried out using the Fisher criterion. The calculated value of the Fisher criterion is determined. The calculated value of the factor being optimized is calculated by substituting the coded values of all columns of the matrix (-1, 0 and +1) of the Y1 equation 3 - table. The values are obtained row by row, not column by column. The calculations for the Y formula are as follows, and the calculation results are included in table 4:

In order to check whether the above regression mathematical model is adequate or not, we determine using the calculated value of the Fisher criterion.

$$F_R = \frac{S_{nad}^2\{Y\}}{S^2\{\overline{Y}\}}$$

where: $S^2\{\overline{Y_1}\} = \frac{\sum_{u=1}^{N} S^2\{Y\}}{N_s - 1}$ $\qquad S_{nad}^2\{Y\} = \frac{\sum_{u=1}^{N-N_s+1}(Y_{Ru} - \overline{Y}_u)^2}{N - N_{k.en} - (N_s - 1)^2}$;

$N - N_{k.en} - (N_s - 1)^2 = 15 - 8 - (3 - 1)^2 = 3$

$N - N_s + 1 = 15 - 3 + 1$=13

Calculate the resulting Y2- encoded values by substituting them into the equation for the charge transfer resistance:

$$S^2\{\overline{Y_1}\} = \frac{\sum_{u=1}^{N} S^2\{Y\}}{N_s - 1} = \frac{18,80}{3-1} = 9,4$$

$$Y_{R2} = -4,48 - 0,6x_1 - 0,5x_2 + 0,3x_3 + 0,2x_1x_2 - 0,25x_1^2 - 0,22x_2^2 - 0,2x_3^2$$

$$Y_{R2.1} = -4,48 + (-0,5) + 0,2 + (-0,25) + (-0,22) = -5,86$$

$$Y_{R2.2} = -4,48 + 0,5 + (-0,2) + (-0,25) + (-0,22) =$$-5,25

$$Y_{R2.3} = -4,48 + (-0,5) + (-0,2) + (-0,25) + (-0,22) =$$-5,05

$$Y_{R2.4} = -4,48 + 0,5 + 0,2 + (-0,25) + (-0,22) =$$ -3,65

$Y_{R2.5}$ =-4,48+0,3+(-0,25)+(-0,2)=-5,23

$Y_{R2.6} = -4{,}48 + (-0{,}3) + (-0{,}25) + (-0{,}2)$ =-5,83

$Y_{R2.7} = -4{,}48 + 0{,}3 + (-0{,}25) + (-0{,}2)$ =-4,03

$Y_{R2.8} = -4{,}48 + (-0{,}3) + (-0{,}25) + (-0{,}2)$ =-4,63

$Y_{R2.9}$ =-4,48+(-0,5)+0,3+(-0,22)+(-0,2) =-5,10

$Y_{R2.10} = -4{,}48 + (-0{,}5) + (-0{,}3) + (-0{,}22) + (-0{,}2)$ =-5,70

$Y_{R2.11} = -4{,}48 + 0{,}5 + 0{,}3 + (-0{,}22) + (-0{,}2)$ =-4,10

$Y_{R2.12} = -4{,}48 + 0{,}5 + (-0{,}3) + (-0{,}22) + (-0{,}2)$ =-4,70

4- table. The calculation results of the values encoded in the equation for adequate dispersion

№	Y_2- Charge transfer resistance			
	Y_{1i}	Y_{1i}	$(Y_{1i}-Y_{R1i})$	$(Y_{1i}-Y_{R1i})^2$
1	-5,755	-5,85	-0,09	0,01
2	-5,155	-5,25	-0,09	0,01
3	-4,955	-5,05	-0,10	0,01
4	-3,555	-3,65	-0,10	0,01
5	-5,155	-5,23	-0,08	0,01
6	-5,655	-5,83	-0,18	0,03
7	-3,855	-4,03	-0,18	0,03
8	-4,555	-4,63	-0,08	0,01
9	-4,905	-5,1	-0,20	0,04
10	-5,605	-5,7	-0,09	0,01
11	-4,005	-4,1	-0,10	0,01
12	-4,505	-4,7	-0,20	0,04

$$\sum_{u=1}^{N-N_s+1} (Y_{R1.u} - \overline{Y}_{1u})^2 = 0{,}203$$

$$S^2_{nad}\{Y_1\} = \frac{0{,}203}{4} = 0{,}051$$

It is known that if the calculated value of the criterion is less than the table value, then that coefficient is adequate and proves that the calculations were carried out correctly.

$$F_{R1} = \frac{S^2_{nad}\{Y\}}{S^2\{\overline{Y}\}} = \frac{0{,}0507}{0{,}015} = 3{,}38$$

$$F_j\left[P_D = 0{,}95;\ f\left\{S^2_{nad}\{Y\}\right\} = 15 - 7 - (3-1) = 5;\ f\{S^2_u\} = 3 - 1 = 2\right] = 4{,}74$$

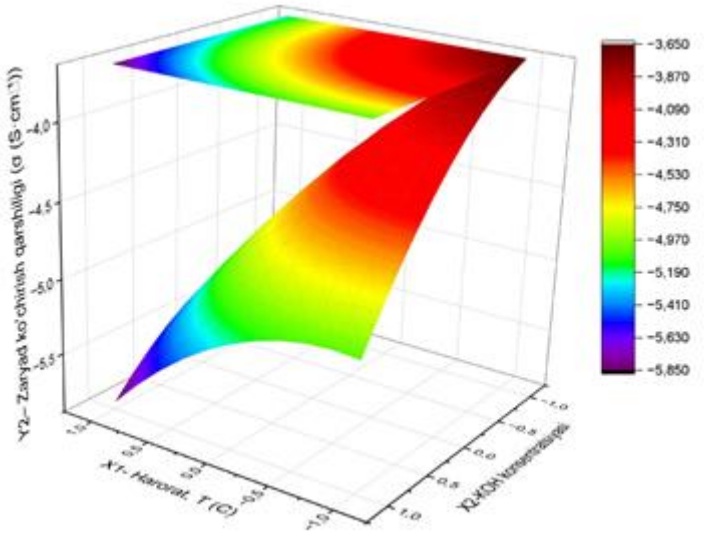
$$F_{R1} = 3{,}38 < 19{,}4 = F_j$$

Therefore, the obtained regression mathematical models represent the studied process with sufficient accuracy.

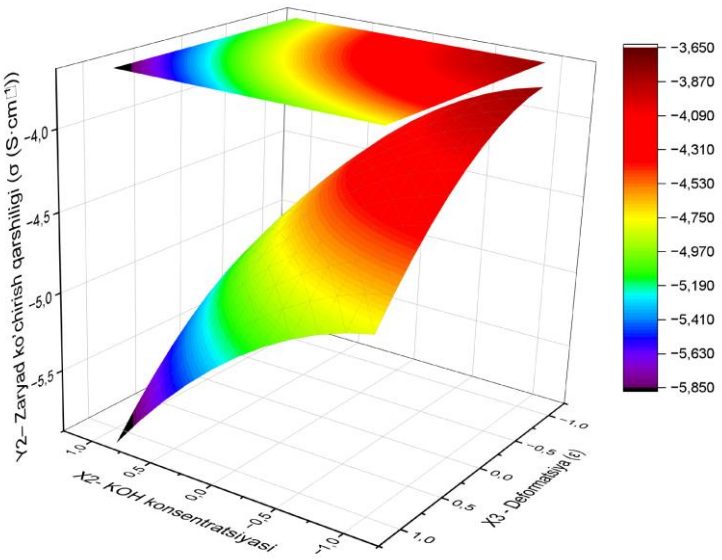
Obtaining optimization solutions and conclusions

Since the equation constructed to determine the characteristics of the output parameter for the study is three-dimensional, one of the input factors in the analysis is assumed to be X_i=0 (central position), and we construct a two-dimensional graph by transforming the models into 3 equations.

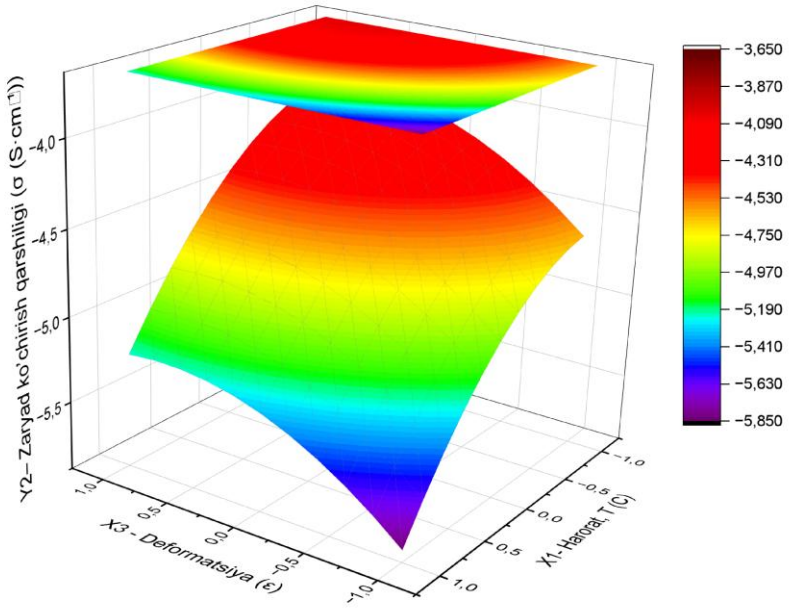
$$Y_{R1} = -4{,}48 - 0{,}6x_1 - 0{,}5x_2 + 0{,}3x_3 + 0{,}2x_1x_2 - 0{,}25x_1^2 - 0{,}22x_2^2 - 0{,}2x_3^2$$



a-figure.



b-figure.



c-figure.

Figure 2. Graph of the model for optimizing the elongation at break of the seam.

Regression analysis of the ionic conductivity showed that the temperature factor has the strongest positive effect on σ. With increasing temperature, the kinetic energy of ions increases and ion transport processes within the electrolyte/membrane are activated. The presence of a linear positive but quadratic negative term for the KOH concentration is found, indicating that this concentration increases σ up to a certain optimal value, after which σ decreases due to increased ion association and viscosity. The positive effect of mechanical deformation is explained by the reorientation of ion channels and the expansion of the active contact area.

The results of the charge transfer resistance showed that increasing temperature and electrolyte concentration accelerate the interface reactions and reduce Rct. At the same time, there is a quadratic effect of the deformation factor, and excessive mechanical loading can lead to disruption of the electrode-electrolyte contact and an increase in Rct. This confirms the need to optimize mechanical and electrochemical factors together when designing flexible electrochemical devices.

The results of the Fisher criterion demonstrate the adequacy of the regression models and prove that the proposed mathematical expressions can describe the experimental results with sufficient accuracy. The obtained models can be effectively used in the prediction and practical design of optimal operating modes for electrochemical systems.

CONCLUSION:

In this work, the dependence of ionic conductivity (σ) and charge transfer resistance (Rct) at the electrode–electrolyte interface on temperature (T), KOH concentration (c), and mechanical deformation (ε) was evaluated using second-order regression models based on central non-compositional design (CCD/MNKT). The significance of the regression coefficients was analyzed using the Student test, and the adequacy of the model was checked using the Fisher test. According to the results, it was observed that for σ, the temperature and deformation factors had a positive effect, while the KOH concentration had a nonlinear (quadratic) effect; this indicates the need for a balance between factors in determining the optimal operating modes. For Rct, it was hypothesized that an increase in temperature and concentration would reduce the resistance due to the acceleration of the interface kinetics, while deformation would increase the resistance after a

certain threshold due to contact/structural deformation. The proposed regression models serve as a practical mathematical tool for predicting σ and Rct for combinations of technological factors and optimizing the energy efficiency of an electrochemical system.

REFERANCE

1. Bard AJ, Faulkner LR. *Electrochemical Methods: Fundamentals and Applications*. Wiley, 2001.
2. Lasia A. *Electrochemical Impedance Spectroscopy and Its Applications*. Springer, 2014.
3. Armand M, Tarascon JM. Building better batteries. *Nature*, 2008.
4. Macdonald JR. Impedance spectroscopy. *Annals of Biomedical Engineering*, 1992.
5. Conway BE. *Electrochemical Supercapacitors*. Springer, 1999.
6. Montgomery DC. *Design and Analysis of Experiments*. Wiley, 2017.
7. Box GEP, Wilson KB. On the experimental attainment of optimum conditions. *JRSS*, 1951.