

Effect of Design Speed in Radius of Horizontal Curves and K of Vertical Curves

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Abstract

Two crucial transitional components in roadway engineering designs are horizontal and vertical curves. The design of these curves is influenced by various elements, the most significant of which is friction, as well as the road's design speeds. In order to give cars on the road a steady rate of revolution, horizontal curves are described as semicircles. The centrifugal force rules of physics are used to calculate the radii of horizontal curves. When a turn is narrower (has a smaller radius) than when it is wider (has a bigger radius), the centrifugal force is higher. Lateral friction (f_s) functions as a force opposing the centrifugal force while the vehicle is driving on a level surface, although it often offers relatively little resistance, necessitating a very large circle for the vehicle to spin on the level. Although the purpose of using a vertical curve is to alter the highway's contour to create a seamless transition between two halted ramps, the vertical curve's design is dependent upon the road's planned design speed. In light of the aforementioned, the study sought to determine the nature of the relationship between design speed and various design criteria, including stopping sight distance (SSD), length of the radius of curvature (R), horizontal sight line displacement (HSO), braking distance (db), maximum span (e_{max}), vertical curvature, lane and shoulder width (L), maximum lateral friction coefficient (f_{max}), as well as the relationship between speed and various engineering design elements, including track width, the relationship between the inferred design speed and the specified design criteria for horizontal and vertical curves. The design speed and the radius of horizontal curves were shown to be positively correlated. To attain the highest level of safety on roads with vertical curves, it is vital to ascertain the value of K while calculating the design speed in both sag and crest vertical curves.

Keywords: design speed, horizontal curve, vertical curve, speed, super-elevation



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1. Introduction

The concept of design speed can sometimes be confusing; some consider it to be the maximum operational speed, while in reality, design speed refers to the target speed, which is critical to achieving safety and influencing behavioural changes by setting clear expectations for drivers as critical determinants of safe vehicle speeds. Traditional practices help set a design speed above the

maximum permissible speed to account for driver errors. Existing design speed guidelines for urban roadways should not be relied upon by those in charge of determining design speeds. According to Davis (2011) [1], planners ought to take the initiative to restrict vehicle speeds, use low turning radius speeds, and limit the number and width of lanes. A proactive strategy uses design to help set and meet a goal speed while directing driver behaviour with both physical and cognitive clues. Reduced curb radii, lane widths, signal sequences, and other speed-management strategies are some examples of these cues. In order to lower vehicle speeds, achieve speed tolerance, and provide pedestrians with greater frontage, street designers must use lower design speeds. In urban areas, the design speed should not exceed 40 km/h, except for some designated lanes [2].

During the period from 1936 to 2001, the concept of design speed underwent a significant change. Barnett (1936) [3] defined the assumed design speed as the quickest driving group's maximum reasonable uniform speed for vehicle operations once they were outside of cities. According to AASHTO (2001) [4] and MUTCD (2000) [5], in 2001, the phrase "design speed" referred to a certain speed that was used to identify the different geometric design elements of the roadway.

It is better to physically separate cars from vulnerable users, such as cyclists and pedestrians, when speeds higher than this are allowed. As long as they are placed between 80 and 100 meters apart, built-in medians or vertical components can serve as barriers. It is advised to employ particular engineering measures, such as signals and traffic congestion management strategies, when traveling more than 200 meters and at speeds more than 40 km/h in order to establish safe circumstances for the use of all forms of transportation [6].

Design speed includes aspects of the cross-section, horizontal alignment and vertical alignment, both explicitly and implicitly. The braking distance (db), vertical curvature, lane and shoulder width (L), maximum lateral friction factor (fmax), stopping sight distance (SSD), length of the radius of curvature (R), horizontal sight line offset (HSO) and maximum overhang emax. These values are determined based on the road's specified design speed, which is considered a part of engineering design [7]. The inferred design speed is the highest speed at which all critical site-specific considerations with respect to speed are met. It is equal to the determined design speed when using limiting or minimum values of design guidelines. Design consistency may or may not be a part of the design speed. The inferred speed is higher and may be linked to higher operational speeds when the engineering design parameters are greater than the minimal values [8].

Operating speeds are influenced by the geometric design features of roads, and there is a strong correlation between speed and a number of geometric design components, including track width, the relationship between the specified and inferred design speeds, and the design requirements for both horizontal and vertical curves [9]. The type and importance of the road, the type and density of traffic, the terrain and geometry of the road, the qualities of the road surface, visibility, and the weather all have an impact on the design speed, according to AASHTO [10].

Curves are a crucial component of road design because they set a top speed that must be followed at all costs, because going over it causes the vehicle to lose control during the bend. When a road's alignment changes from up to down (or vice versa) or from right to left (or vice versa), curves are positioned. The two primary curve types that are required on highways are horizontal and vertical curves. To facilitate the use of the road infrastructure and lower hazards, the proper safety precautions must be taken at all horizontal and vertical bends [11].

Curves or circular arcs used to align or alter a road's direction are known as horizontal curves. The steeper the curve becomes and the smaller its radius, the more dangerous it gets. The most important thing in designing any horizontal curve is the distance for stopping which should be adequate and safe. Horizontal curves are of many types, namely spiral or transitional curve, simple curve, compound curve and reverse curve [12].

Unlike horizontal curves that are circular, vertical ones are parabolic. They change the slope of the road either symmetrically or asymmetrically. The main design criteria are to establish the safe sight distance for passing and the appropriate degree of sight distance at the curve's highest point [13]. The length of curve must also be sufficient to provide a safe sight distance when stopping. The Vertical curves are of two types: Apex curve and Doubling curves, in the apex curve the road alignment is shifted from uphill to downhill, while doubling curve is shifted downhill to uphill. Horizontal curves are considered as not only very important part of road design but also are a large portion in terms of causing problems for vehicles during turning [14]. Besides the centrifugal force, which allows the vehicle to follow a curved line (the road), there is also the gravitational force that attracts it towards the ground. The centrifugal force is less in a smooth wide turn having a greater radius than in a sharp turn with lesser radius. Sed 2. Hence, turning requires the car to revolve in a verifiably enormous circle; to counter this issue, streets should be constructed with a slight bit of slope while forming flat curves, making turns less demanding [15].

2. Mathematical relationships between design speed and both horizontal curves and vertical curves

The relationship between the design speed and both horizontal and vertical curves has been explained by a number of mathematical theories. One of the most crucial of these models is the point mass model, which establishes the minimal radius of curvature as a function of the designated design speed, particularly when designing horizontal curves. It also establishes the road's maximum height and maximum coefficient of lateral friction. The relationship between the designated design speed and the AASHTO geometric design criteria for both horizontal and vertical curves is demonstrated by the following ratios:

$$R_{min} = \frac{V^2}{15(0.01 e_{max} + f_{max})}$$

Where: R_{min} = Minimum bend radius (ft/m)
 V = Design speed (mph or km/h)
 e_{max} = Maximum road elevation factor
 f_{max} = Maximum coefficient of lateral friction

The minimal vehicle speed is an additional criterion for the geometric design of vertical and horizontal bends. This is the distance a driver must observe an obstacle on the road in front of their car, which would necessitate a full stop or response. There are two components to the minimum vehicle speed:

1. The distance covered during perception and response
2. The distance covered when braking
3. The formula for braking distance on a level, vertical slope

$$d_b = 1.075 \frac{V^2}{a}$$

Where: d_b = Braking distance (feet or meters)
 a = Deceleration rate (ft/s² or m/s²)

The braking distance is adjusted according to the following equation in cases where there is a vertical slope:

$$d_b = \sqrt{30}[(a/32.2) \pm G]$$

Where: G = (Height/Run: ft/ft or m/m)

Braking distance is included as part of the SSD along with the distance traveled during perception and reaction according to the following equation:

$$SSD = 1.47 Vt + 1.075 \frac{v^2}{a}$$

where the brake reaction time is t (2.5 seconds).

AASHTO states that when designing the horizontal curve HSO, the length of the inner side of the curves—which may obstruct the optical view—is taken into account, as shown in the following equation.

$$HSO = R [1 - \cos(\frac{28.65S}{R})]$$

Where: S = Stopping sight distance (ft or m)

R = Curve radius (ft or m)

Road portions where the tangent grade changes should have vertical curves. Depending on the design speed and whether the curve is crest or sag, the radii of the vertical curvature should not be smaller than the values listed in Table 1. Road sections where the tangent grade changes should have vertical curves. Depending on the design speed and the kind of curve (crest or sag), the radii of vertical curvature should not be smaller than the values listed in Table 1. The radius of the crest type of vertical curve may be lowered to 1000 m on a road with a design speed of 60 km/h in an urban area; the lengths of vertical curves shall not be shorter than the values shown in Tables 1 and Table 2.

Table 1. Radius of vertical curvature

Design speed (km/h)	Radius of vertical curvature	
	Crest type	Sag type
100	6.500	3.000
80	3.000	2.000
60	1.400	1.000
50	800	700
40	450	450
30	250	250
20	100	100

Table 2. length of vertical curve

Design speed (km/h)	length of vertical curve
100	85
80	70
60	50
50	40
40	35
30	25
20	20

In general, vertical curves should be easy to apply and produce a safe design where larger curves are preferred where practical. A vertical curve can be identified by two measures: the length of the

curve and the radius of curvature, which can be roughly compared to a parabolic curve. The relationship between two metrics is described by the following equation:

$$L_r = \frac{A}{100} R$$

Where: R = radius of vertical curvature (m)

L_r = length of vertical curve (m)

A = algebraic difference in grades (%)

This connection states that a radius of curvature can be employed more readily than curve length and is roughly equal to the value of dividing the distance between two points by the algebraic difference in grades and a constant of K.

The following is an expression for the length of curvature:

$$L_r = K^a A$$

Comparing the equation: $K^a = R/100$

$$R = 100 K^a$$

The parabola is commonly employed in vertical curves because it provides a consistent rate of change in slope, which helps the driver maintain a steady viewing distance. It is also simple to calculate by hand. The rate of change of slope per unit length in a basic parabola is constant Q and may be written as follows:

$$Q = \frac{A}{L}$$

Where: L = The vertical curve's length is measured horizontally

A = The algebraic difference in grades, expressed as a percentage

K is the vertical curve's length (m) for a 1% grade change (m/unit%).

Q is the rate at which grade changes per unit length (%/m).

One straightforward method for figuring out the size of the vertical curve is to use the idea of the value of K. The length of the curve, L, for all values of A, is determined by a single value of K for each design speed and sight distance. A circular curve of radius R, which closely resembles a parabolic curve by the relation, can also be used to form a vertical curve.

$$R = 100 K$$

The horizontal length of the vertical curve (L) for a given sight distance (L_s) is calculated by the following expressions for two cases:

1. When L is less than L_s :

$$L = (2 L_s) - (L/A)$$

2. When L is more than L_s :

$$L + (L_s^2) A/C$$

Where: L = Horizontal length of the vertical curve (m)

L_s = the sight distance (m)

A = Algebraic difference between vertical grading (%)

C = Sight line constant

When the vertical curve parameter K is substituted for L/A , the result is:
 $(L_s^2)/C = K$

The K value for the following circumstances can be determined using the equations above:

1. K value for halting sight distance and summit bends when L
2. When $L > L_s$, the K value for summit bends and stopping sight distance
3. When $L < L_s$, the K value for summit curves and passing sight distance
4. When $L > L_s$, the K value for summit curves and passing sight distance
5. K value for stopping sight distance and sag curves when $L < L_s$

3. Results and Discussions

1. Effect of Design Speed in Radius of Horizontal Curves

In contrast to straight, level roadways that offer a clear sight line for extended distances, horizontal curves pose a considerable issue when evaluating the safe sight distance. As a result, visual distance limitations are frequently accommodated by lowering the permitted design speed. The allowable radius of the horizontal curve can be calculated using the design speed, coefficient of friction, and allowable super-height on the curve.

There are two ways to determine the allowable sight distance for a horizontal curve: either the sight distance is shorter than the curve's length, or the sight distance is longer than the curve's length. Based on the geometric characteristics, there is a special formula for calculating the sight distance for each case.

Given the length of the curve L , the radius R_v and the sight distance S , the distance at which the sight obstruction can occur from the inner edge of the road M_s can be calculated from the following equation:

$$S < L: R_v (1 - \cos 2865S/R_v)$$

$$S > L: R_v (1 - \cos 2865L/R_v) + (S - L/2) \sin (2865L/R_v)$$

For example, when the design speed is 100 km/h on a horizontal curve, the superelevation is (e) set to 5% and the lateral friction coefficient (f_s) is 0.9, the horizontal curve radius that ensures safe vehicle operation is determined as follows:

$$R = v^2 / g(e + f_s) = 100 (1000/3600)^2 / 9.8 (5 + 0.9) = 593 \text{ meter}$$

To calculate the effect of the design speed on the radius of the horizontal curve, apply the following equations:

$$R_{\text{ruling}} = v^2 / (e + f)g \quad R_{\text{min}} = v'^2 / (e + f)g$$

V = design speed

V' = minimum design speed

f = coefficient of friction

e = rate of super-elevation

At all regions, the maximum value of " e " is taken as 0.07 while " e " value is taken as 0.1 at hill roads. At horizontal curves, the allowed speed is calculated by considering coefficient of lateral friction as the following:

$$e + f = 0.08 + 0.14 = 0.22$$

The safe allowable speed calculated by:

$$V_a = \sqrt{0.22gR} \text{ m/sec.}$$

It is noted that at speeds higher than the design speed, transverse vehicle skidding occurs, requiring additional width for safety purposes, especially when the vehicle passes a horizontal curve and follows a path with a shorter radius at the front wheels.

Assuming the road has n lanes, R is the radius of the curve and I is the wheelbase length of the longest vehicle, the mechanical width (W_m) of the road is given by:

$$W_m = nI^2/2R$$

It was proposed a relation between physiological widening at horizontal curves: $W_{ps} = V/9.5\sqrt{R}$

Where: V = design speed (km/h)

R = radius of the curve (m)

According to the tables and curves contained in the AASHTO reports, which are relied upon to determine the values of the friction coefficient according to the design speeds of the road, these tables can be used to determine the smallest radius of the horizontal curve directly from the following equation and Tables (3 - 5)

$$R = \frac{V^2}{127(e+f)}$$

Figure 1 shows the relation between super elevation and radius of curve.

Table 3. Relation between design speed and maximum relative gradient and equivalent maximum relative slope at both metric system and U S customary

Metric			U.S. Customary		
Design speed km/h	Maximum Relative Gradient %	Equivalent Maximum Relative Slope	Design speed mph	Maximum Relative Gradient %	Equivalent Maximum Relative Slope
20	0.80	1:125	15	0.78	1:128
30	0.75	1:133	20	0.74	1:135
40	0.70	1:143	25	0.70	1:143
50	0.65	1:154	30	0.66	1:152
60	0.60	1:167	35	0.62	1:161
70	0.55	1:182	40	0.58	1:172
80	0.5	1:200	45	0.54	1:185
90	0.47	1:213	50	0.50	1:200
100	0.44	1:227	55	0.47	1:213
110	0.41	1:244	60	0.45	1:222
120	0.38	1:263	65	0.43	1:233
130	0.35	1:286	70	0.40	1:250
			75	0.38	1:263
			80	0.35	1:286

Table 4. The relation between design speed and maximum radius at both the metric system and U S customary system

Metric		U.S. Customary	
Design speed km/h	Maximum Radius m	Design speed mph	Maximum Radius ft
20	24	15	114
30	54	20	203
40	95	25	317
50	148	30	456
60	213	35	620
70	290	40	810
80	319	45	1025
90	480	50	1265
100	592	55	1531
110	716	60	1822
120	852	65	2138
130	1000	70	2479
		75	2846
		80	3238

Table 5. The relationship between the design speed and the minimum horizontal curve radius

Speed design (km/h)	Minimum curve radius according to superelevation (m)		
	6%	7%	8%
120	710	670	630
110	600	560	530
100	460	440	420

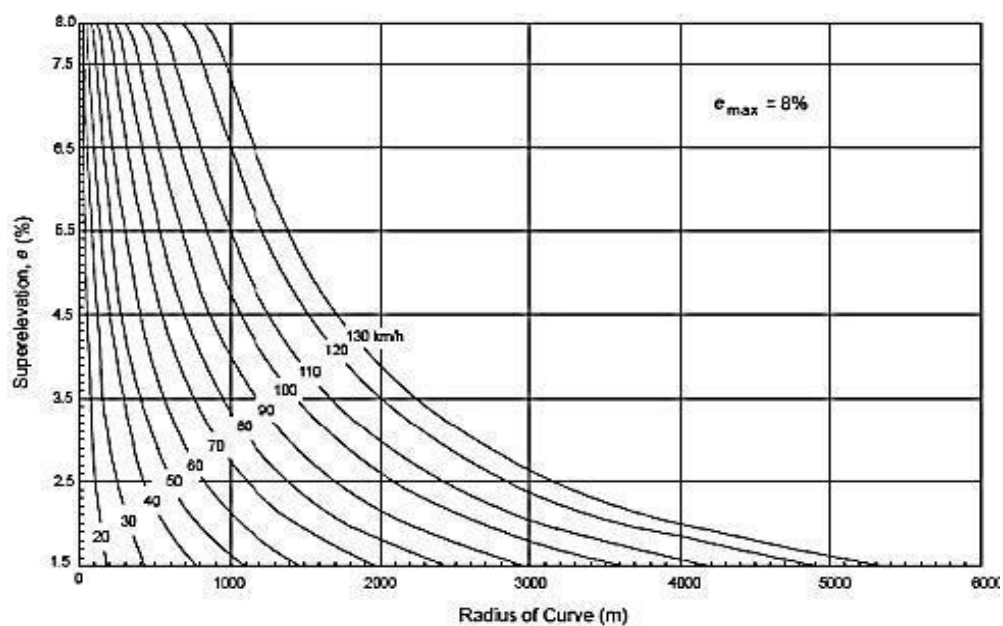


Figure 1. The relation between super-elevation and radius of curve

2. Effect of Design Speed in K of Vertical Curves

Vertical curves may have a limited effect on very slight changes in gradient, while short vertical curves have a significant effect on road conditions. Therefore, the K value for vertical curves should be much higher for very slight changes in gradient for reasons of minimum visibility. This is of great importance on high-quality roads, especially for sagging curves.

In normal road design situations, the vertical acceleration resulting from the transition from one slope to another is limited to a maximum of 0.05 g, where g is the acceleration due to gravity (9.8 m/s). However, at intersections and on low-speed roads, a vertical acceleration of 0.10 g can be used when necessary. When crossing a parabolic vertical curve at a constant speed, the vertical component of the vertical acceleration at the curve is given by the following equation:

$$a = \frac{V^2}{12960K}$$

Where: a = vertical component of radial acceleration (m/sec²)

V = speed (km/h)

K = a measure of vertical curvature

Table 6 shows the K values for the vertical acceleration and design speeds specified as eigenvalues for sage curve design

Table 6. K values for vertical acceleration and specific design speeds for sage curve design

Design Speed (km/h)	K Values	
	a = 0.05 g	a = 0.10 g
40	2	1.4
50	3	2
60	5	3
70	7	4
80	9	5
90	11	7
100	15	8
110	18	9
120	22	11

Sight distance limits typically apply to a vertical curve's minimum length. The design speed of the horizontal alignment determines the absolute minimum sight distance required for vertical curves on all roadways. However, as a matter of good design practice, vertical curves longer than the minimal length should be employed wherever practicable. The sight distance requirements for creating vertical apex curves are displayed in Table 7.

Table 7. K Values for Crest Vertical Curve Design

Design Speed (km/h)	Stopping Requirements			Minimum Overtaking Requirements	
	Stopping Sight distance	K h ₁ = 1.05 m h ₂ = 0.20 m C = 433	1.4 x Stopping Sight Distance	Intermediate Sight Distance (2 x SSD) (m)	K h ₁ = 1.05 m h ₂ = 1.15 m C = 879
40	40	4	55	80	7
50	55	7	76	110	13
60	70	12	106	150	25
70	90	20	132	185	42
80	110	32	160	245	60
90	135	45	195	280	88
100	165	66	237	345	133
110	210	101	295	420	200
120	245	143	350	500	283

4. Conclusions

For horizontal curves on roads, it is necessary to take design criteria into account when determining the design speed on roads so that an appropriate minimum radius for the horizontal curve can be proposed. This ensures a reliable impact of speed limits on road safety. The current research demonstrated models for design speed on horizontal curves by using speed data to analyze the operating speed. The results indicated that the square root of the radius is a key parameter in determining the operating speed of vehicles based on the design speed, given that horizontal curves have higher collision rates than straight sections or paths similar to horizontal curves in length and traffic composition. This is evident at radii less than 1000 meters, and collision rates increase significantly at radii less than 200 meters. This is consistent with the significant correlation between the differences in the curve diameter and the design speed. The larger the curve radius, the higher the design speed, which means that there is a positive effect of the design speed on the radius in horizontal curves.

In the case of vertical curves, it is necessary to specify the value of K when calculating the design speed, as this value represents the horizontal distance over which the slope changes by 1% on the vertical curve. Therefore, speed tables and design standards provide the minimum target limits for the deflection coefficient K.

For general safety in vertical curves, the sag vertical curve must be of sufficient length so that the light beam distance is approximately equal to the stopping sight distance. Therefore, stopping sight distances must be used for different design speeds. As in the case of crest vertical curves, when determining the design speed, the vertical curvature rate K must be equal to the length of the vertical curve L divided by the algebraic difference in tangent degrees (A) as a percentage ($K = L / A$).

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