SemantJournals

American Journal of Technology Advancement 2997-9382, Vol 03, No. 03, 2026

Article

Mathematical Modeling of Sprinkler Irrigation Process Using Stationary Machines

Avlakulova Mohigul Meyliyevna[1]

Independent researcher, Karshi State Technical University, Uzbekistan

E-mail: mavlakulov@mail.ru

Abstract: Sprinkler irrigation technology is widely used for efficient irrigation of agricultural crops. One of the key requirements for achieving high agronomic efficiency is the uniform distribution of water across the irrigated field. Uneven water distribution may lead to excessive irrigation in some areas and insufficient irrigation in others, which negatively affects crop productivity and water use efficiency. Therefore, the development of scientifically based methods for analyzing and optimizing the operation of sprinkler irrigation systems is an important task in modern agricultural engineering. This study is devoted to the development of a mathematical model for the irrigation process using stationary sprinkler devices. The mechanism of water distribution over the field surface was analyzed based on probability density functions describing the spatial distribution of droplets produced by rotating sprinkler heads. In the proposed model, the angular distribution of water flow was assumed to be uniform, while the radial distribution of droplet flight distance was described using a normal probability distribution. The modeling approach also takes into account the hydraulic characteristics of sprinkler nozzles and the geometric arrangement of irrigation devices. As a result of simulation calculations, the spatial distribution of irrigation depth across the field was obtained. Based on the calculated data, the coefficient of variation of irrigation depth, the uniformity coefficient, and indicators of effective irrigation were determined. The modeling results made it possible to evaluate the influence of the distance between sprinkler devices and their operating parameters on irrigation uniformity.

Keywords: Sprinkler Irrigation, Mathematical Modeling, Water Distribution Uniformity, Stationary Sprinkler Systems, Irrigation Efficiency, Agricultural Water Management, Probability Distribution Modeling.

This is an open-access article under the CC-BY 4.0 license

1. Introduction

Efficient use of water resources in agricultural production is one of the most important challenges. In regions with arid and semi-arid climates, irrigation technologies play a key role in ensuring stable crop yields. Currently, extensive scientific research is being carried out to improve irrigation technologies and reduce water consumption.

Sprinkler irrigation is distinguished by its ability to distribute water relatively uniformly over the field area. However, in practice the uniformity of water distribution in sprinkler systems depends on many factors. The nozzle diameter, operating pressure, rotation speed of the apparatus, spacing

between devices, and wind conditions significantly influence the uniformity of water distribution. Therefore, both theoretical and practical investigation of sprinkler irrigation processes is of great importance.

Designing irrigation machines experimentally requires considerable labor and financial resources. For this reason, the use of mathematical modeling methods in the analysis of irrigation systems is considered effective. Mathematical models allow the operating modes of sprinkler devices to be analyzed in a virtual environment.

In recent years, numerous studies have been conducted on modeling sprinkler irrigation processes. Researchers such as Kravchenko, Chernovolov, Keller, Zhang and others have developed various models for evaluating water distribution uniformity [1], [2], [3], [4]. In addition, studies using artificial intelligence and machine learning methods for analyzing irrigation systems have also appeared [5], [6], [7]. The objective of this research is to mathematically model the water distribution process of stationary sprinkler devices and to evaluate irrigation uniformity.

2. Materials and Methods

Designing, testing and adapting rotating sprinkler devices require significant labor, time and financial resources. Therefore, the use of automated design systems makes it possible to improve irrigation machines and reduce overall costs.

This article analyzes programs intended for mathematical modeling of water distribution across the field through sprinkler devices that are part of stationary irrigation systems or irrigation machines operating by positional method.

If the displacement of the device during one cycle of impulse motion is much smaller than the size of the irrigation zone, the rotational motion of the apparatus is assumed to be uniform. In such a case, the device operates within a full circle or a certain sector.

Deflector nozzles operate under low pressure conditions, which reduces the amount of energy required to distribute water across the field. At the same time, they provide relatively uniform rainfall distribution [8], [9].

However, the use of sector nozzles leads to a reduction in the irrigation zone area, which increases rainfall intensity. If the soil water infiltration capacity is low, it is necessary either to increase the movement speed of the machine or reduce the water discharge supplied to stationary nozzles.

Reducing water discharge decreases the overall productivity of irrigation equipment, which is not considered appropriate from a practical point of view.

Optimization of the operating regime of irrigation equipment based on the criteria of uniform liquid distribution makes it possible to reduce material and energy consumption and increase the probability of achieving high-quality irrigation under variable operating conditions such as field relief or wind influence. Such optimization is advisable to perform using mathematical modeling methods [10].

Uniform water distribution across the field is primarily evaluated using the coefficient of variation of irrigation depth Fq. If deviations in irrigation depth are considered as random variables with a known distribution law, the coefficient of variation can be determined through functional relationships with irrigation efficiency coefficients.

The advantage of using the coefficient of variation is that almost all automatic control systems and standard computer programs include ready-made functions for its calculation.

Currently, sprinkler machines such as DSHK-64A “Volzhanka” and DF-120 “Dnepr”, widely used in CIS countries, operate in positional mode.

The DSHK-64A type wheeled pipeline is equipped with medium-range sprinklers with nozzle

diameters of 7 and 8 mm arranged in a row along the pipeline. Depending on the nozzle diameter, the distance between positions is 18 or 24 meters.

The DF-120 irrigation system operates through hydrants located at a distance of 54 meters from each other. The pipeline of this machine is equipped with wings with a length of 27 meters, at the ends of which medium-range sprinkler devices are installed.

During irrigation, the arrangement scheme of sprinklers can be represented as a rectangle with dimensions **A×B**, where: A – distance between sprinklers on the pipeline B – distance between positions or wings.

Since stationary sprinkler systems have similar design schemes, water distribution modeling for them can be carried out using programs developed for positional irrigation machines.

In this case, experimental data on water distribution along the radius of the irrigation zone are sufficient for program operation, while other parameters are calculated by simulation.

During the simulation process, the rotation of the apparatus is assumed to be uniform. Since the displacement during one cycle of impulse motion is very small relative to the irrigation zone dimensions, overlapping irrigation zones result in an overall water distribution corresponding to continuous uniform rotation of the sprinkler device.

3. Results and Discussion

Rotating stationary sprinkler devices form a circular rainfall pattern on the field surface. The area irrigated by a single sprinkler device is called the **irrigation zone**. The mathematical model of the process is constructed under the following assumptions: the distribution of rainfall along the circular angle is expressed through the probability density f(α) and the total flow rate Q, where the flow density per unit angle is qα=Q·f(α); the droplet flight distance is distributed along the radius with probability density f(ϱ); the water discharge Q is assumed to be constant; wind speed is assumed to be zero; the installation height of the sprinklers is constant; and the field surface is considered horizontal.

The uniform distribution of water across the field is evaluated using the following indicators: the coefficient of variation of irrigation depth qF, the effective irrigation coefficient, the irrigation deficit coefficient, and the excessive irrigation coefficient. The device is assumed to be located at the origin of the XOY coordinate system (Fig. 1) and operates along a circular path. The position of the elementary area dF is described in both Cartesian and polar coordinates. The use of two coordinate systems is necessary because the apparatus rotates along the angular direction while the water jet is projected along the radius, whereas irrigation uniformity is evaluated over the field area represented in Cartesian coordinates.

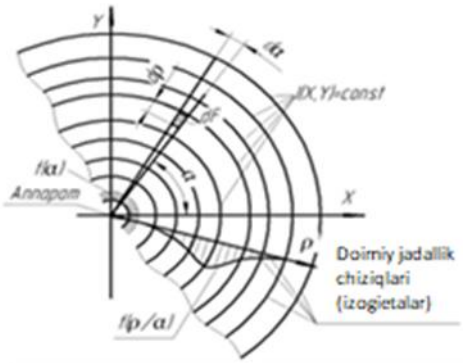

Figure 1. Scheme for modeling water distribution by a sprinkler device [1], [2].

The model is based on field research data and simulates water distribution in sprinkler irrigation. It combines the **Richards equation** and probabilistic models. Calculations were performed under windless conditions (wind speed assumed to be zero) and taking into account the high evaporation characteristic of the Kashkadarya region. The model is based on **HYDRUS-2D**, in which the Richards equation and **van Genuchten parameters** were calibrated using inverse modeling [9], [12].

The probability of liquid falling on an elementary area dF, separated by intervals dα in the angular direction and dϱ in the radial direction within the irrigation zone, is equal to the product of the probabilities corresponding to these angular and radial intervals. Thus, the droplet movement and its distribution in the irrigation process can be expressed through probability density functions using the following fundamental equation [11], [12], [13].

For uniform rotation:

$$P(dF) = f(\alpha)\, d\alpha\, f(\rho)\, d\rho,$$

where f(α)=1/(2π) (for smooth rotation); $q(\alpha) = Q \cdot f(\alpha)$; Q is the water flow rate (L/s).

f(ϱ) - normal distribution:

$$f(\rho) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\rho - M)^2}{2\sigma^2}\right),$$

Where ϱ–droplet flight distance, M – mean flight distance (for the conditions of Kashkadarya region 11–15 m), σ – standard deviation (3.2–5 m).

This formula is used to calculate the radial distribution of droplets under windless conditions. However, considering the high evaporation in the Kashkadarya region due to high temperatures, evaporation losses are assumed to be 10–20% [14], [15], [16]. For example, for the Rosa-1 sprinkler with M=11, σ=3,2, at ϱ=10 f(ϱ)≈0,125. Under our conditions, using low-pressure nozzles and sprinklers increases the uniformity coefficient to 80–90%.

The rainfall intensity at a given point of the field is defined as the ratio of the amount of water falling on that point per unit time to the area of that point. When the variables ϱ and α are considered independent random variables, rainfall intensity can be calculated as follows:

$$i(\rho, \alpha) = \frac{Q \cdot f(\alpha) \cdot f(\rho)}{\rho\, d\rho\, d\alpha}$$

In the SI system, rainfall intensity is expressed in kg/(m²·s), which is equivalent to millimeters of rainfall per second used in irrigation practice. For any area irrigated by a stationary sprinkler device, the irrigation depth is determined by multiplying the rainfall intensity by the operating time T:

$$m = i \cdot T\,, \qquad (2)$$

where T – operating time of the device (hours), i – rainfall intensity.

Considering the climatic conditions of the Karshi district, where evaporation reaches 5–8 mm/day, rainfall intensity can be reduced by 0,8–0,9, since deflector nozzles distribute water evenly under low pressure [17]. For example, with Q=1,2 L/s and T=3600 s, the average irrigation depth per irrigation is approximately: m≈22 mm.

For soils with medium mechanical composition and low infiltration capacity in the Kashkadarya region, optimization of rainfall intensity makes it possible to reduce excess water from 0,4 to 0,2.

The Cartesian coordinates of the elementary area dF are related to polar coordinates through the following relationships.

$$\begin{cases} X = \rho \cdot \cos\alpha \\ Y = \rho \cdot \sin\alpha \end{cases} \qquad (3)$$

When the sprinkler operates over a full circle, the rotation of the reactive device is assumed to be uniform. Therefore, all angular values have equal probability and the probability density over the angle is:

$$f(\alpha) = \frac{1}{2 \cdot \pi}. \qquad (4)$$

For a single-nozzle sprinkler, the droplet flight distance along the radius approximately follows a normal distribution:

$$f(\rho) = \frac{1}{\sigma_\rho \sqrt{2 \cdot \pi}} \exp\left(-\frac{(\rho - M_\rho)^2}{2 \cdot \sigma_\rho^2}\right), \qquad (5)$$

where ϱ – droplet flight distance, σϱ – variance of droplet distance, Mϱ – expected value of droplet distance.

For double-nozzle sprinkler devices, the probability density of droplet flight distance is described as:

$$f(\rho) = \frac{C_1}{\sigma_{\rho 1} \sqrt{2 \cdot \pi}} \exp\left(-\frac{(\rho - M_{\rho 1})^2}{2 \cdot \sigma_{\rho 1}^2}\right) + \frac{C_2}{\sigma_{\rho 2} \sqrt{2 \cdot \pi}} \exp\left(-\frac{(\rho - M_{\rho 2})^2}{2 \cdot \sigma_{\rho 2}^2}\right), \qquad (6)$$

Where C_1, C_2 – weighting coefficients, Mϱ1, σϱ1 – statistical parameters for droplets from the first nozzle, Mϱ2, σϱ2 – parameters for droplets from the second nozzle.

These parameters are selected during modeling to ensure the most uniform water distribution across the irrigation zone.

Below is an example of mathematical modeling of water distribution by a sprinkler device (Fig. 2). Based on the technical parameters of the device, the following values are used for modeling:

Mϱ=12 m, σϱ=3,2 m, Q=1,2 l/s

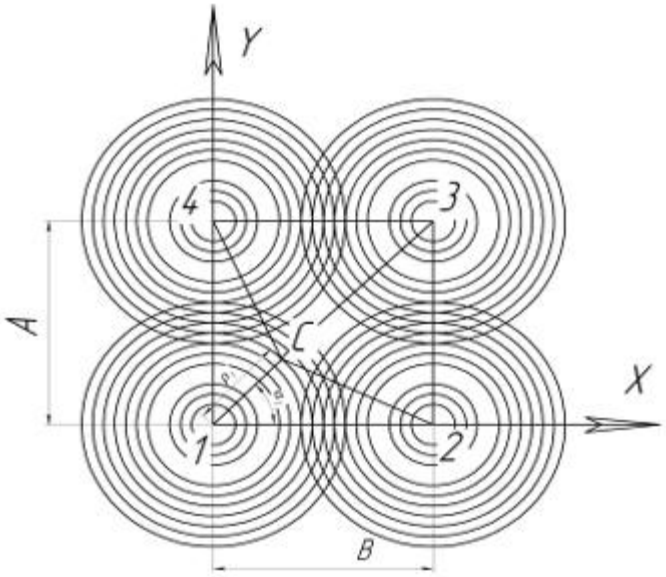

Figure 2. Layout scheme of the sprinkler apparatus.

The first sprinkler device is assumed to be located at the origin of the XOY coordinate system. The second device is located along the X-axis at distance B. The coordinates of the third device are

(B, A), and the fourth device is located at point (0, A).

To improve the accuracy of modeling results, the geometric dimensions of the overall irrigation zone are determined and irrigation depths for all four devices are calculated.

For this purpose, coordinates of points C(X,Y) are defined with a step of one meter and represented in matrix form. By changing the distance between the devices, attempts are made to select parameters that ensure the most uniform water distribution across the field. However, in practice, this problem is quite complex.

The uniformity of water distribution within a rectangular area of size A and B is quantitatively evaluated using the irrigation depth matrix. The coordinate boundaries of X and Y are determined within a square area bounded by lines connecting the sprinkler devices and divided into one-meter sections. Irrigation depths are calculated at the center of each elementary cell (Fig. 3).

As a result of calculations, a matrix of irrigation depths MD with dimensions A×B is obtained. This matrix is used to determine variation coefficients as well as minimum, average, and maximum irrigation depths for different values of parameters A and B. Under calm atmospheric conditions it is logical to assume A=B. Only in sector irrigation cases these parameters differ.

Then the boundary values of effective irrigation depths are calculated:

$$q_{f\,min} = q_{fsr} - 0.25 \cdot q_{fsr}; \quad q_{f\,max} = q_{fsr} - 0.25 \cdot q_{fsr};$$

Variation series are constructed based on irrigation depth matrices. On this basis, effective, excessive, and insufficient irrigation coefficients are calculated (Fig. 3). The central boundary of the variation series corresponds to the average irrigation depth qf_{sr}. The left boundary corresponds to the minimum irrigation depth qf_{min}, and the right boundary corresponds to the maximum irrigation depth qf_{max}.

The boundary values are then expanded to the right with a step of $0{,}25qf_{sr}$ until they exceed the largest irrigation depth in the matrix. Similarly, on the left side, the boundaries are extended until the interval corresponding to the minimum irrigation depth class is reached.

The average irrigation depth is calculated directly from the irrigation matrix and compared with the accepted calculated value.

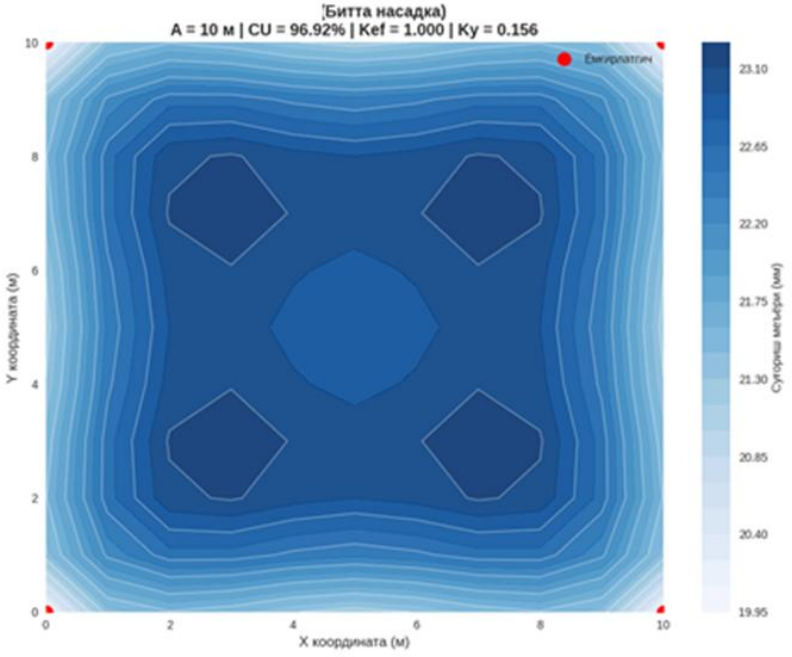

Figure 3. Irrigation depth program and contour graph for a single sprinkler device.

The results of the conducted studies showed that achieving sufficiently uniform water distribution across the field using single-nozzle stationary sprinkler devices is quite difficult. Under such conditions, the irrigation efficiency coefficient was significantly lower than the agronomic requirements. At the same time, excessive and insufficient irrigation coefficients were approximately two times higher than the agronomic norms.

When A=B=25 m, excessive irrigation zones were observed in overlapping areas of sprinkler coverage (Fig. 3). The average irrigation depth across the field was 4,8 kg/m². The central part of the diagram as well as the areas directly under the sprinkler devices showed zones of excessive water accumulation. In addition, excessive irrigation was observed along lines connecting the centers of sprinkler devices both horizontally and vertically.

Particularly, the square area formed by connecting the central points of sprinkler devices shows high practical interest for analyzing irrigation uniformity.

During modeling we used **HYDRUS-2D**, based on the **Richards equation** and inverse modeling calibration of van Genuchten parameters.

To evaluate modeling reliability, the Christiansen uniformity coefficient (CU) and effective irrigation coefficient (K_{ef}) were used.

$$CU = 100\left(1 - \frac{\sum |q_i - \bar{q}|}{n\bar{q}}\right),$$

$$Kef = \frac{\sum q_i \cdot \mathbb{I}(q_i \geq 0.75\bar{q})}{n\bar{q}}$$

Here: CU – uniformity coefficient. For nozzle spacing of 25–30 m, CU reached about **80%**, while in our results, due to reduced spacing, it increased to 92–97% (Fig. 4).

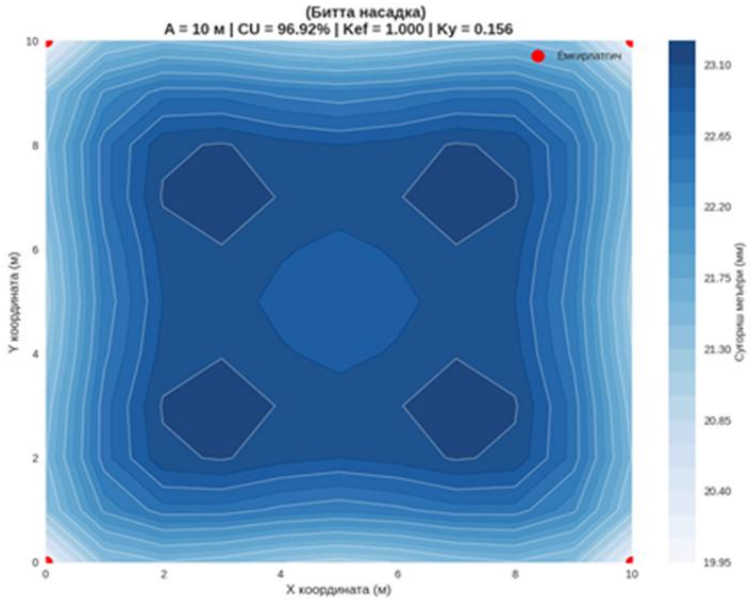

Figure 4. Indicators and calculation of uniform water distribution for a single sprinkler device (A=B=10 m).

Kef – effective irrigation coefficient. When Kef = 0,75, it represents areas with above-average irrigation depth. Under our conditions Kef ≥ 0,997, which increases water saving efficiency by 30–50%. The optimal distance between positions **A** should be less than 13,4 m, meaning 10–12 m, where CU ≥ 80%. From the MD matrix, the average irrigation depth was: m≈22 mmю Using CU calculations and considering low soil infiltration in the Kashkadarya region, reducing the spacing to 10–12 m produced the best results.

4. Conclusion

The results of the study demonstrated the possibilities of mathematical modeling of water distribution processes using stationary sprinkler devices. Using the developed model, irrigation depths and their spatial distribution across the field were determined.

The obtained results are important for improving irrigation systems and ensuring efficient use of water resources in agriculture.

REFERENCES

[1] L. V. Kravchenko, *Modeling and Optimization of Sprinkler Irrigation of Agricultural Crops with Front-Acting Machines*. Zernograd, 2003.

[2] L. V. Kravchenko and V. A. Lukhanin, *Modeling the Sprinkler Irrigation Process with Front-Acting Machines with Sector Nozzles*. KubSAU, 2014.

[3] V. A. Chernovolov and L. V. Kravchenko, "Optimization of nozzle spacing in front-acting irrigation machines," 2014.

[4] V. A. Chernovolov, "Calculation of the efficiency indicators of sprinkling with single-jet devices," 2015.

[5] V. A. Chernovolov and L. V. Kravchenko, "Mathematical modeling of liquid distribution processes in agricultural technologies," 2016.

[6] N. K. Murodov and M. Avlakulov, "Analysis of the thermal regime of the soil during irrigation of agricultural crops by the subirrigation method," *New University. Series: Technical Sciences*, no. 1, pp. 12–16, 2016.

[7] M. Avlakulov, "Multilayer capillary models for developing a stochastic pattern of mass transfer," in *Proc. 1st Int. Sci. Pract. Conf. European Scientific Congress*, Madrid, Spain: Barca Academy Publishing, 2023, p. 187.

[8] Y. Li *et al*., "Simulating coefficient of soil moisture content uniformity of sprinkler irrigation systems using a COMSOL-3D model," *Agricultural Water Management*, vol. 301, p. 108399, 2024.

[9] A. A. Al-Gaadi *et al*., "Prediction of water distribution uniformity of sprinkler irrigation system based on machine learning algorithms," *Scientific Reports*, vol. 13, p. 21567, 2023.

[10] A. A. Al-Gaadi *et al*., "Modeling water distribution uniformity of medium-sized sprinklers using artificial neural networks," *Journal of Agricultural Engineering*, vol. 7, no. 2, p. 41, 2025.

[11] L. Zhang *et al*., "Modeling the sprinkler water distribution uniformity by data-driven methods based on effective variables," *Agricultural Water Management*, vol. 215, pp. 63–73, 2019.

[12] F. Dechmi *et al*., "Development and validation of a modified model to simulate the sprinkler water distribution," *Computers and Electronics in Agriculture*, vol. 115, pp. 1–10, 2018.

[13] R. K. Srivastava *et al*., "Assessing sprinkler irrigation uniformity using a ballistic simulation model," *Agricultural Water Management*, vol. 84, pp. 89–100, 2022.

[14] R. Chen *et al*., "Coupled weather and crop simulation modeling for smart irrigation planning: A review," *Water Supply*, vol. 24, no. 8, pp. 2844–2865, 2024.

[15] T. Bonny *et al*., "Towards a modelling, optimization and predictive control framework for smart irrigation," *Heliyon*, vol. 10, no. 14, p. e34126, 2024.

[16] J. Keller and R. Bliesner, "Variable-rate irrigation uniformity model for linear-move sprinkler systems," *Transactions of the ASABE*, vol. 64, no. 4, pp. 1123–1135, 2021.

[17] Merriam and J. Keller, "Christiansen uniformity revisited: Re-thinking uniformity assessment in rainfall simulator studies," *Catena*, vol. 220, p. 106410, 2022.