

Study of the Effect of Porous Medium Deformation on the Filtration Characteristics of an Oil Reservoir

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Abstract:

This study investigates the impact of oil reservoir skeleton deformation on the main filtration and hydrodynamic characteristics during multiphase fluid flow. To describe the process, a mathematical model was used that accounts for changes in the pressure, porosity, and absolute permeability of the deformable porous medium. The numerical solution of the system of equations was performed using the large particle method. The spatio-temporal changes in the reservoir parameters were analyzed for various values of the skeleton compressibility coefficient and distances between production wells. It was established that a decrease in reservoir pressure leads to the deformation of the pore space and a reduction in porosity and permeability. It was shown that as the compressibility coefficient increases, the deformation effects intensify, and as the distance between production wells decreases, their hydrodynamic interaction increases, which accelerates the changes in the filtration characteristics in the inter-well region. The obtained results confirm the necessity of accounting for the deformability of the porous medium when modeling oil field development.

Keywords: multiphase filtration, deformable porous medium, oil reservoir, porosity, permeability, pressure, compressibility coefficient, large particle method.

Introduction

Deformation processes in oil and gas reservoirs represent one of the major factors determining the changes in the porosity, permeability, and hydrodynamic characteristics of a formation during its development. The extraction of formation fluids is accompanied by a decrease in pore pressure, resulting in an increase in the effective stress acting on the solid framework of the porous medium. Consequently, changes occur in the volume of the pore space, the structure of pore channels, and the

dimensions of pore throats, which in turn directly affect porosity, absolute and relative permeability, flow rate, and well productivity. The theoretical foundation for describing the coupled processes of porous medium deformation and fluid flow was established in the classic work of M. Biot [1], which formulated a three-dimensional theory of consolidation for a saturated porous medium. Subsequently, the principles of poroelasticity theory were developed as applied to the mechanics of oil and gas reservoirs and modern problems of hydromechanical coupling [2, 3].

Some of the first experimental studies showed that the influence of external pressure on the filtration properties of a rock can be significantly stronger than its influence on porosity. In the study by Fatt and Davis [4], a decrease in sandstone permeability was established as overburden pressure increased. Later, Fatt [5] showed that pore volume compressibility is not only an individual characteristic of a specific rock but also depends on pressure. Experiments by Dobrynin [6] also confirmed the sensitivity of sandstone petrophysical properties to effective stress. These studies demonstrated that the assumption of constant porosity and permeability, often used in classical flow models, can lead to significant deviations when modeling the development of deformable reservoirs.

The physical mechanism of changes in filtration properties is primarily related to the deformation of pores and pore throats. When reservoir pressure decreases, the load in the "skeleton–fluid" system is redistributed toward the solid skeleton, causing its compaction. As a result, the characteristic dimensions of the flow channels decrease. At the same time, even a relatively small change in total porosity can be accompanied by a significant decrease in permeability. Gangi [7] proposed a physical model linking the change in permeability of porous and fractured rocks to effective pressure. Studies by Davies and Davies [8] have shown that the sensitivity of permeability to stress is determined by the geometry of the pore space and differs significantly for highly porous, weakly cemented sandstones and low-permeability, tight sandstones. Thus, the dependence of permeability on stress cannot be considered a universal characteristic and must be determined taking into account the structure of a specific reservoir.

It is of particular importance to account for the spatial distribution of stresses and changes in formation properties in the vicinity of production wells. Han and Dusseault [9] developed a flow model for the near-wellbore zone, taking into account the dependence of porosity and permeability on the stress state. The authors demonstrated that pressure depletion during production alters effective stresses and causes spatially heterogeneous changes in flow characteristics. The most pronounced changes can be observed near the wellbore, where pressure and stress gradients are particularly high. This is fundamentally important for evaluating the well flow rate, as a decrease in permeability specifically in the bottomhole zone can significantly increase flow resistance and reduce fluid inflow.

Experimental studies also show that deformational changes significantly depend on the loading path. Nguyen et al. [10] examined the compaction of uncemented sand and weakly cemented sandstone under various stress paths. It was established that

mechanical deformation and permeability changes are nonlinearly related and differ for the elastic, brittle, and plastic stages of deformation. The results of triaxial tests and computed tomography presented by Hu et al. [11] further demonstrated that as deviatoric stress increases, the rock initially compacts, leading to a decrease in porosity, and upon further loading, dilatancy may occur. The change in permeability is determined not only by the magnitude of porosity but also by the configuration of pore channels, microcracks, and the tortuosity of flow paths. Consequently, the relationship between porosity and permeability in a deformable reservoir is generally nonlinear.

From the perspective of oil field development, the influence of deformation on multiphase flow is of particular interest. Xiong et al. [12] investigated changes in the petrophysical characteristics of weakly cemented sandstone during its compaction and showed that an increase in effective pressure is accompanied by a decrease in porosity, absolute permeability, and pore throat dimensions. Moreover, the reduction in permeability significantly exceeded the relative change in porosity. In oil displacement by water experiments, it was also established that relative permeability curves change, and residual oil saturation and irreducible water saturation increase. Consequently, the deformation of the rock framework affects not only the absolute flow capacity of the formation but also the nature of the concurrent flow of individual phases. This circumstance is particularly important when modeling water-oil systems, as changes in the structure of the pore space can directly affect the oil recovery factor and well water cut dynamics.

To quantify such effects, coupled hydromechanical models are increasingly being used. Settari and Mourits [13] showed that the simultaneous solution of flow and geomechanical equations yields results that differ significantly from calculations that do not account for mechanical changes in the formation. Geertsma [14] previously demonstrated a link between pressure depletion, reservoir compaction, and surface subsidence. In later models, changes in permeability are directly linked to reservoir pressure, effective stress, or volumetric strain. Thus, To and Chang [15], comparing models with constant and variable permeability, demonstrated the necessity of accounting for the coupled "pressure–stress–permeability" process when describing depleting sandstone reservoirs. Similar conclusions have been drawn for low-permeability oil reservoirs, where the sensitivity of permeability to effective stress can have a particularly strong impact on production dynamics [16].

Modern research continues to clarify the microstructural mechanisms of this interaction. It has been established that the stress sensitivity of permeability is largely determined by changes in the volumes of large pores and pore throats and can be well described by nonlinear, particularly exponential, dependencies on effective stress [17]. Alongside this, methods of numerical modeling of pore networks and three-dimensional visualization of rock structure make it possible to establish a relationship between macroscopic filtration characteristics and local deformation of the pore space [11, 17]. Fundamental works on oil reservoir mechanics also note the need for the

simultaneous consideration of effective stresses, pore pressure, skeleton compressibility, and changes in transport characteristics [18].

Thus, an analysis of existing studies shows that the deformation of the oil reservoir rock matrix is one of the key factors driving the dynamic changes in its flow and storage properties. A decline in reservoir pressure leads to an increase in effective stress and the deformation of the pore structure, which is accompanied by changes in porosity and, to a significantly greater extent, permeability. For multiphase systems, changes in relative permeabilities and residual saturations acquire additional significance. Therefore, during the mathematical modeling of oil field development processes, it is advisable to treat porosity and permeability not as constant parameters, but as functions of pressure or effective stress. This approach makes it possible to more accurately account for the mutual influence of deformation and flow processes and to evaluate their impact on pressure distribution, flow velocities, and the performance of production wells.

Formulation of the problem

The mathematical model of the process is given in the following form:

- mass conservation equation for oil phase

$$\frac{\partial}{\partial t}(ms_1) + \frac{\partial}{\partial x}(u_1) = 0, \quad (1)$$

- mass conservation equation for water phase

$$\frac{\partial}{\partial t}(ms_2) + \frac{\partial}{\partial x}(u_2) = 0, \quad (2)$$

- porosity change dependency

$$m = m_0 + \beta_m(p - p_0), \quad (3)$$

- Darcy's law

$$u_1 = \frac{Kk_1}{\mu_1} \frac{\partial p}{\partial x}, \quad u_2 = \frac{Kk_2}{\mu_2} \frac{\partial p}{\partial x}, \quad (4)$$

- dependency of permeability

$$K = K_0 \left(\frac{m}{m_0} \right)^d, \quad (5)$$

Let us add to the system of equations the obvious equality

$$s_1 + s_2 = 1, \quad (6)$$

- dependencies for viscosities

$$\mu_1 = \text{const}, \quad \mu_2 = \text{const} \quad (7)$$

- and relative permeabilities for phases

$$k_1 = \begin{cases} \left(\frac{1-s_2}{1-s_{2E}} \right); & s_2 > s_{2E}, \\ 1 & s_2 \leq s_{2E} \end{cases} \quad (8)$$

$$k_2 = \begin{cases} \left(\frac{s_2 - s_{2E}}{1 - s_{2E}} \right); & s_2 > s_{2E}, \\ 0 & s_2 \leq s_{2E} \end{cases} \quad (9)$$

To derive the equation for pressure, we sum the equations (1) - (2). As a result, we obtain

$$\frac{\partial}{\partial t} [m(s_1 + s_2)] + \frac{\partial}{\partial x} (u_1 + u_2) = 0.$$

or, considering (6) and (3)

$$\beta_m \frac{\partial p}{\partial t} + \frac{\partial}{\partial x} (u_1 + u_2) = 0.$$

Ultimately, the pressure equation will take the form

$$\beta_m \frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \left[\left(\frac{Kk_1}{\mu_1} + \frac{Kk_2}{\mu_2} \right) \frac{\partial p}{\partial x} \right] \quad (10)$$

Providing initial

$$s_1(x,0) = s_1^0, \quad s_2(x,0) = s_2^0, \quad p(x,0) = p^0 \quad (11)$$

and boundary conditions

$$\left. \frac{\partial p}{\partial x} \right|_{x=0} = 0, \quad p(l_1, t) = p_1^*, \quad p(l_2, t) = p_2^*, \quad \left. \frac{\partial p}{\partial x} \right|_{x=L} = 0, \quad (12)$$

where l_1 – first well coordinate, l_2 – second well coordinate, L – second well coordinate, p_1^* – pressure in the first well, p_2^* – pressure in the second well.

To solve the problem (1) - (12), we use the large particle method. To do this, we will first introduce an Euler grid.

$$\Omega_{th} = \left\{ t_{j+1} = t_j + \tau, \quad j = 0, \dots, R-1; \quad j = \frac{T}{\tau}; \right. \\ \left. x_{i+1} = x_i + h, \quad i = 0, \dots, N-1; \quad i = \frac{L}{h} \right\} \quad (13)$$

where h – grid pitch in space, τ – grid step in time, N – number of nodes in space, R – number of nodes over time, L – distance in space, T – maximum considered time. The environment is modeled by a system of liquid particles that coincide at a given moment with the cell of the Euler grid. We divide the calculation of each time step into three stages:

Step 1 - ignore the effects associated with the movement of the elementary cell and approximate equation (10) at time t_j

$$\beta_m \frac{p_i^{\sim} - p_i^k}{\tau} = \frac{1}{h^2} \left\{ b_{i+\frac{1}{2}}^k \cdot p_{i+1}^{\sim} - \left[b_{i+\frac{1}{2}} + b_{i-\frac{1}{2}} \right]^k \cdot p_i^{\sim} + b_{i-\frac{1}{2}}^k \cdot p_{i-1}^{\sim} \right\} \quad (14)$$

where

$$b_i^k = \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_i^k$$

$$A_i \cdot p_{i-1}^{\sim} - B_i \cdot p_i^{\sim} + C_i \cdot p_{i+1}^{\sim} = -F_i \quad (15)$$

where

$$A_i = \frac{\tau}{\beta_m h^2} b_{i-\frac{1}{2}}^k, \quad B_i = 1 + \frac{\tau}{\beta_m h^2} \left\{ b_{i-\frac{1}{2}}^k + b_{i+\frac{1}{2}}^k \right\}, \quad C_i = \frac{\tau}{\beta_m h^2} b_{i+\frac{1}{2}}^k, \quad F_i = p_i^k$$

To solve the system of equations (15), we apply the driving method. The driving coefficients are calculated using the following formulas:

$$\begin{cases} \alpha_{i+1} = \frac{C_i}{B_i - A_i \alpha_i}, & i = \overline{1, N} \\ \beta_{i+1} = \frac{F_i + A_i \cdot \beta_i}{B_i - A_i \alpha_i}, & i = \overline{1, N} \end{cases} \quad (16)$$

and the pressure according to

$$p_i^{\sim} = \alpha_{i+1} p_{i+1}^{\sim} + \beta_{i+1}. \quad (17)$$

From the boundary conditions, we have

$$\alpha_1 = 1, \quad \beta_1 = 0, \\ p_N^{\sim} = p^*.$$

Step 2. Having determined the pressure values, we calculate the phase velocities.

$$(u_1)_{i+\frac{1}{2}}^{\sim} = - \left(\frac{Kk_1}{\mu_1} \right)_i^{\sim} \frac{p_i^{\sim} - p_{i+1}^{\sim}}{h}, \quad (u_2)_{i+\frac{1}{2}}^{\sim} = - \left(\frac{Kk_2}{\mu_2} \right)_i^{\sim} \frac{p_i^{\sim} - p_{i+1}^{\sim}}{h}$$

Step 3. from the conservation laws, we find

$$\begin{aligned} m_i^{k+1} &= m_0 + \beta_m (p_i^{\sim} - p_0) \\ (s_1)_i^{k+1} &= \frac{1}{m_i^{k+1}} \left[m_i^{k+1} (s_1)_i^k + \frac{(u_1)_{i-1/2}^{\sim} - (u_1)_{i+1/2}^{\sim}}{h} \tau \right] \\ (s_2)_i^{k+1} &= 1 - (s_1)_i^{k+1} \\ (k_1)_i^{k+1} &= \begin{cases} \frac{(1 - (s_2)_i^{k+1})}{(1 - s_{2E})} & (s_2)_i^{k+1} > s_{2E} \\ 1 & (s_2)_i^{k+1} \leq s_{2E} \end{cases}, \end{aligned}$$

$$(k_2)_i^{k+1} = \begin{cases} \left(\frac{(s_2)_i^{k+1} - s_{2E}}{1 - s_{2E}} \right), & (s_2)_i^{k+1} > s_{2E} \\ 0 & (s_2)_i^{k+1} \leq s_{2E} \end{cases}$$

$$K_i^{j+1} = K_0 \left(\frac{m_i^{j+1}}{m_0} \right)^d$$

$$K_i^{j+1} = K_0 [1 - a_K (p_0 - p_i^*)]$$

$$K_i^{j+1} = K_0 e^{a_K (p_0 - p_i^{j+1})}$$

After obtaining the parameter values, we proceed to the next time layer, etc.

Numerical solution

To investigate the stress-strain state of a saturated porous medium, computational experiments were conducted using the following parameter values:

$m_0 = 0,23$, $K_0 = 10^{-15} m^2$, $(s_1)_0 = 0,6$, $(s_2)_0 = 0,4$, $(s_1)_0 = 0,6$, $(s_2)_0 = 0,4$, $p_0 = 10^6 Pa$, $p^0 = 10^6 Pa$, $p^* = 5 \cdot 10^5 Pa$, $\mu_1 = 2,23 \cdot 10^{-3} Pa \cdot s$, $\mu_2 = 10^{-3} Pa \cdot s$, $\beta_m = 10^{-8} Pa^{-1}$, $a_K = -1,25 \cdot 10^{-6} Pa^{-1}$, $t = 1 year$

Figure illustrates the dynamics of reservoir pressure at points located at various distances from the production well, for different values of the skeleton compressibility coefficient β_m . It can be seen that the pressure decreases over time, with the most rapid drop observed near the well. As the distance from the well increases, the pressure declines more slowly. Furthermore, an increase in the skeleton compressibility coefficient leads to a faster decrease in pressure, which is associated with a more pronounced deformation of the porous medium during reservoir development.

The figure shows the change in the porosity of the medium over time for various values of the skeleton compressibility coefficient β_m . The most noticeable decrease in porosity is observed near the production well, where the drop in reservoir pressure and the deformation of the skeleton are more pronounced. As the distance from the well increases, the change in porosity becomes less intense. Additionally, an increase in β_m heightens the sensitivity of the pore space to pressure changes and leads to a more significant reduction in porosity.

Figure 3 presents the dynamics of changes in the absolute permeability of the medium at a point located 20 m from the production well, for different distances between two wells (120, 180, and 240 m). It can be seen that as the distance between the wells decreases, the absolute permeability declines more rapidly, which is associated with increased mutual hydrodynamic interference between the wells and a more significant drop in reservoir pressure in the interwell zone. As the distance between the wells increases, this effect weakens, resulting in a slower decrease in permeability. Furthermore, an increase in the skeleton compressibility coefficient β_m

leads to a more pronounced decrease in permeability, which is due to a stronger deformation of the pore space as the pressure drops.

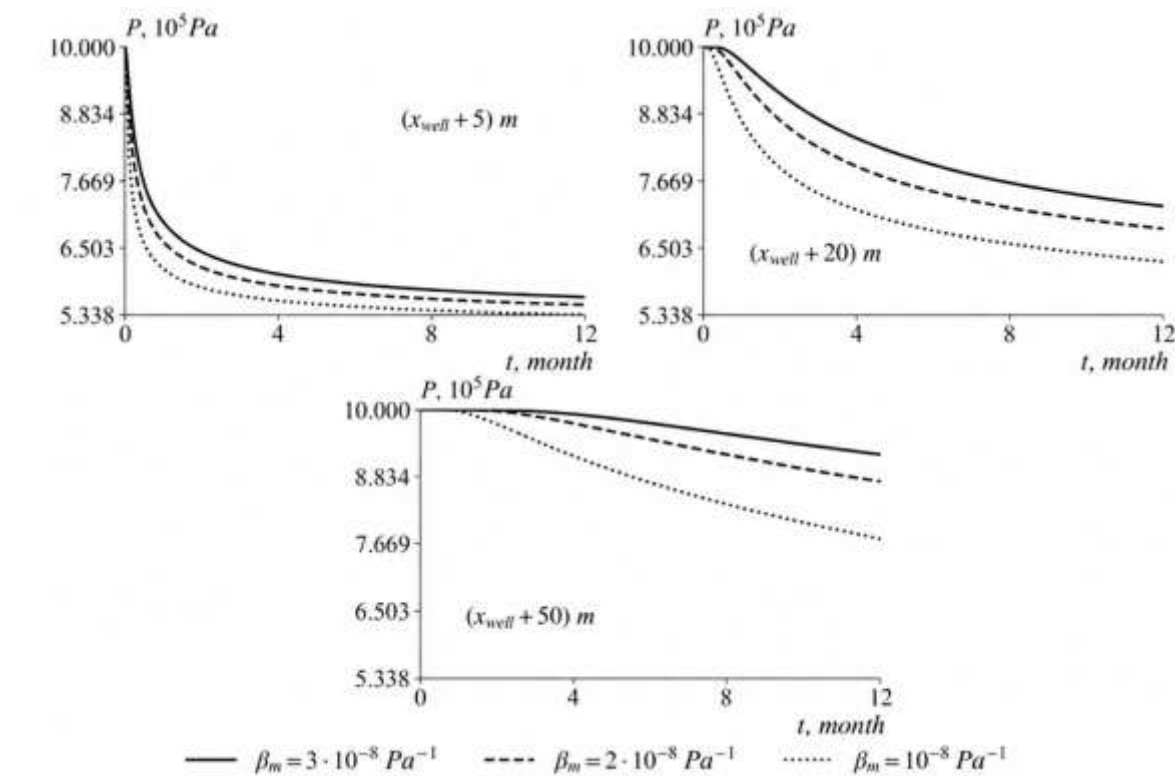


Figure 1. Influence of medium compressibility on pressure change dynamics

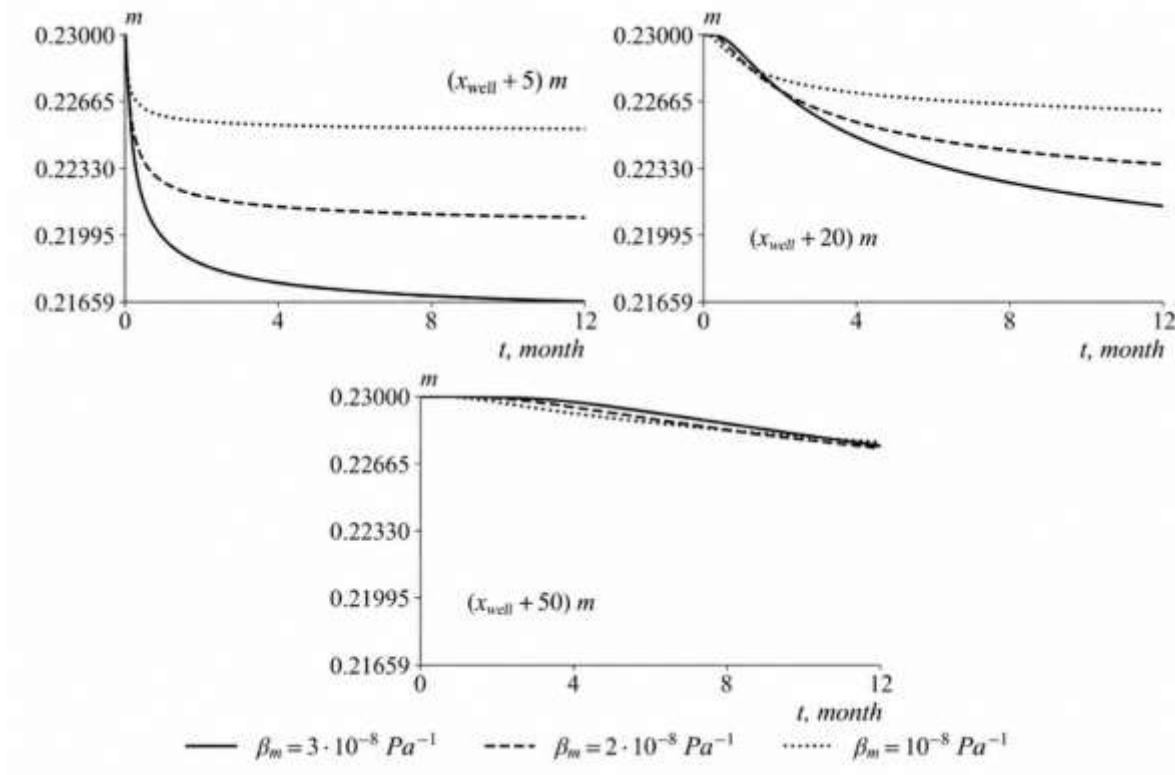


Figure 2. Influence of medium compressibility on the dynamics of porosity change

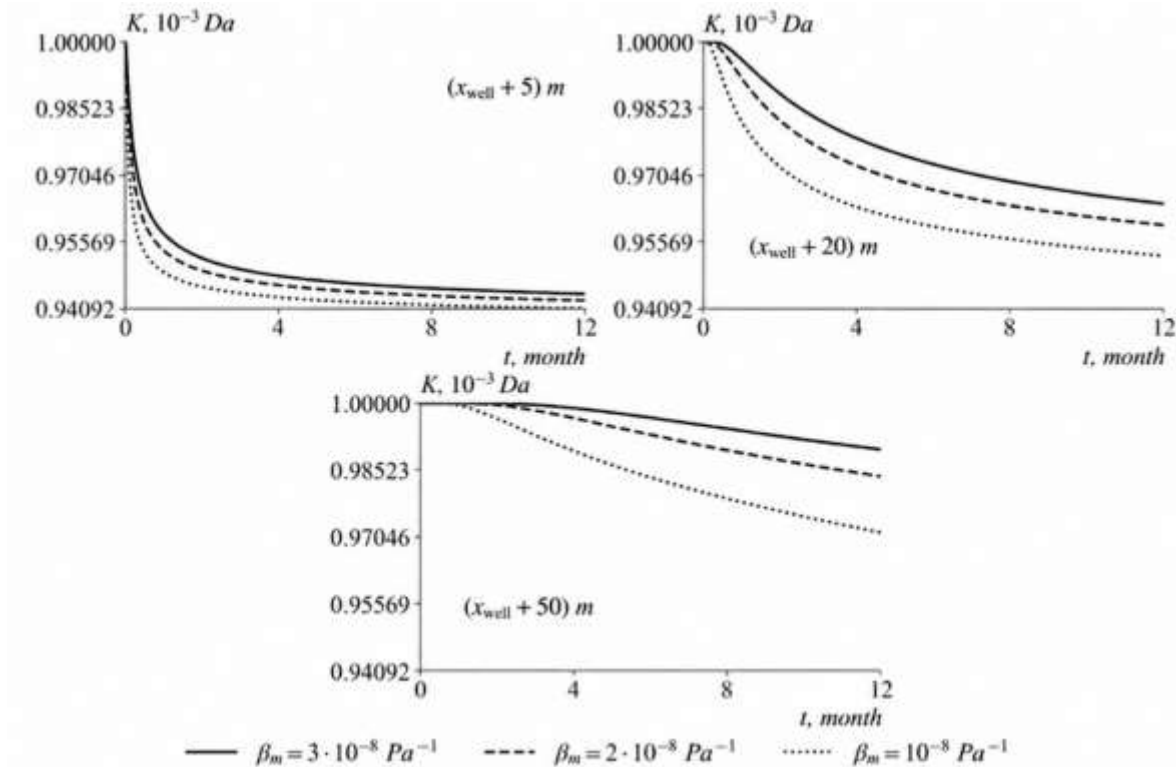


Figure 3. Influence of medium compressibility on the dynamics of permeability change

Conclusion

In this study, based on a multiphase mathematical model, the influence of oil reservoir skeleton deformation on the main filtration and hydrodynamic characteristics of the medium was investigated. For the numerical solution of the system of equations, the large-particle method was used, allowing for the tracking of spatiotemporal changes in pressure, porosity, and absolute permeability during the operation of production wells. The obtained results showed that a decrease in reservoir pressure is accompanied by the deformation of the pore space, which consequently reduces the porosity and permeability of the reservoir. It has been established that the extent of these changes significantly depends on the skeleton compressibility coefficient: as it increases, the deformation effects become more pronounced. Furthermore, it has been shown that the mutual influence of production wells increases as the distance between them decreases, leading to a more rapid decline in pressure and filtration characteristics in the inter-well region. The conducted calculations confirm the necessity of accounting for the deformability of the porous medium when modeling multiphase filtration in oil reservoirs, as neglecting changes in porosity and permeability can lead to a less accurate assessment of field development dynamics. The proposed approach can be used to analyze and forecast filtration processes in deformable oil reservoirs under various reservoir parameters and well placement schemes.

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